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AN EXPERIENCE ORIENTED MASTERY LEARNING STRATEGY IN GRADE NINE ALGEBRA

- AN EXPERIMENTAL STUDY

by

Ⓒ JAMES H. JEFFREY

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
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EDMONTON, ALBERTA
SPRING, 1980

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "An Experience Oriented Mastery Learning Strategy in Grade Nine Algebra - An Experimental Study", submitted by James H. Jeffrey in partial fulfilment of the requirements for the degree of Master of Education.

Alton T. Olson
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Date April 23, 1980

ABSTRACT

The primary purpose of the study was to develop a mastery learning strategy and to investigate the effects of the strategy on students and their learning. The effect that a number of student characteristics (e.g., aptitude, prior achievement, attitude toward mathematics and self-concept of ability) had on student achievement was examined. In turn, the effect which achievement had on these student characteristics was investigated.

A secondary purpose of the study was to determine by means of classroom observations, the effect that the mastery learning strategy had on cooperation among students as opposed to competition in their learning.

Following the identification of the essential elements of a mastery learning strategy from the review of the literature, a unit of mathematical content was selected and basic learning task objectives were specified for the unit. The criteria for mastery were made explicit for the basic learning tasks, the subunits and the entire unit of content.

The six Grade 9 classes of one school experienced various sequences of the mastery and non-mastery treatments during a period of four weeks. The total number of students was 153 and two teachers were involved.

At the beginning of the experiment all the students wrote a series of attitude and self-concept of ability tests (pretests). Following the administration of the summative test the students wrote

the identical series of attitude and self-concept of ability tests (posttests). Two sets of achievement scores were obtained by means of the summative and postsummative tests. Analysis of variance, *t*-tests, Pearson product-moment correlations, multivariate analysis and stepwise multiple regression analysis procedures were carried out on the data collected to test the hypotheses of the study. No statistical analysis was performed on the data obtained by classroom observations; thus, no statistically valid result was derived regarding the cooperation aspect of the experiment.

The results of the study were as follows:

1. The tenets of mastery learning theory identified in the review of the literature were, in general, supported. D. Drost, in his companion report, provides an extensive account of this finding.
2. Teacher Evaluation of Achievement and Sequential Test of Educational Progress (STEP) variables played the most important predictive role in the mastery learning situation.
3. The School and College Ability Test (Non-Verbal) variable played the most important predictive role in the non-mastery learning situation.
4. Although it was not possible to derive a statistically valid answer to the question concerning cooperation under mastery learning conditions, it appeared that a larger number of tutorings occurred in the non-mastery learning situation compared with the mastery learning situation.

In general, the students favoured the mastery approach over the non-mastery method of teaching the unit of algebra. They were of the opinion that their lessons were more consistent with the needs of the individual, more interesting and enjoyable, and organized in such a way as to facilitate learning. Nevertheless, the majority of students felt that more time should have been spent during the initial instruction of each subunit and during the course itself.

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CHAPTER I

THE PROBLEM

INTRODUCTION

Over the years, the public school has had to endure a great deal of criticism, much of it undeserved, but some, at least, justified. Charles E. Silberman (1970), in expressing his indignation at the failures of the public school, condemned the schools for being "grim, joyless places". He stated that what is mostly wrong with the schools is due, not to venality or indifference, or stupidity, but to "mindlessness" which he defined as, "The failure or refusal to think seriously about educational purpose, the reluctance to question established practice".

Bloom (1968) expressed the opinion that the most wasteful and destructive aspect of our present educational system is the set of expectations about student learning which each teacher brings to the beginning of a new course or term. The teacher expects that a third of the students will learn well what is taught, a third will learn less well, and a third will fail or "get by" at best. In time this pernicious prophecy is fulfilled and in so doing results in unfortunate consequences. The aspirations of the teacher are reduced; the attitude of the unsuccessful student toward learning and his desire for further learning are diminished; the ego and self-concept of such a student are subjected to continual depreciation so that he suffers frustration and humiliation through many of his school years; the young person often becomes alienated from both school and society.

Educators dare not be guilty of mindlessness, dare not fail to challenge this established practice; the costs to the individual and to society are too great.

Bloom (1968) has expounded a mastery approach to student learning which he contends will provide almost all students with the successful and rewarding experiences which, at present, are enjoyed by merely a few. Bloom (1973a, 1973b) has claimed that by employing a mastery learning approach in a group-based classroom almost all students can master what is taught; they learn more material in less time; they display a greater interest in and attitude toward the subject learned; and they are motivated for further learning. As the students receive many assurances of their competence or success they are provided with powerful incentives toward establishing positive attitudes and self-concepts.

A teacher who endeavours to implement such a mastery approach to student learning could scarcely be accused of mindlessness. In fact, it may be claimed that by his efforts he proves that he is very mindful of the needs of his students. He is concerned, not only with the cognitive aspects of learning but also of the affective consequences.

Many practitioners in the field believe that teaching is a complex matter and that the commonly group-based classroom situation is not conducive to a mastery learning approach. However, according to Carroll (1971) teaching should be a simple matter for a teacher who knows his trade. He states that,

> Although we are not sure of every facet of the learning process, we do have extensive knowledge of the fundamentals. It is a known fact that students who are confident and well-motivated, possess good basic intelligence and aptitude, and are provided with good instruction, can learn a fantastic amount in relatively short periods of time. Furthermore, what is well-learned tends to be well-retained, especially if it is periodically reviewed and tested. It is also known that some pupils learn much faster than others but, notwithstanding, nearly all pupils can learn what they are supposed to learn in school, given enough time.

If teaching is viewed as a process concerned with the management of learning it becomes a relatively simple matter. Carroll believes that the teacher's function is:

- to specify what is to be learned;
- to motivate students to learn it;
- to supply instructional materials and to administer them at a rate suitable for each pupil;
- to monitor pupil progress;
- to diagnose and remedy difficulties;
- to supply praise and encouragement for good performance; and
- to provide opportunities for review and practice to ensure retention over lengthy periods of time.

For many years education was looked upon as a selection process. The "fittest", who survived, in time graduated and formed the professional elite. The "failures" departed from the educational scene at the appropriate point and occupied their niche in society, often in menial and rather demeaning occupations. Today, many countries demand more education for their citizens. More and more, students are continuing in school to complete at least a high school diploma program. The student body is now much more heterogeneous, both in aptitude and motivation, than in the past.

The increasing heterogeneity of students poses many problems with regard to curriculum development and methodology. For example, practitioners must seek a variety of programs and instructional modes to ensure that the greatest number of students will be equipped to enable them to function adequately in a complex society. However, students differ greatly in how they learn. According to Bloom (1968), some students learn well by means of independent learning experiences while others require highly structured situations. Perhaps individual student characteristics (e.g., intelligence, attitude, self-concept) have a significant bearing on the type of learning situation under which a student may achieve success.

This report, along with a companion report prepared by Dale Drost, presents the results of a project in which a mastery learning strategy was developed and carried out in a school to examine the claims stated above and to seek answers to the questions posed.

STATEMENT OF THE PROBLEM

The main purpose of this joint study was to develop a mastery learning strategy for teaching a segment of the Grade 9 algebra course and to investigate the effects of the strategy on students and their learning.

Dale Drost addressed himself to questions regarding differences in achievement between the mastery and non-mastery treatment groups; the relationship between student aptitude and attainment of mastery criterion; and the effect on the efficiency of the student's learning over the series of subunits into which the unit of algebra was divided.

This part of the joint study concerns itself mainly with the effect of a number of student characteristics on their learning and, in turn, the effect of the students' learning upon these student characteristics.

The student characteristics include both cognitive characteristics and affective characteristics (e.g., attitude, self-concept). With respect to this purpose the following questions were raised:

Question I

What differences exist in student characteristics between the students who attain the achievement criterion and those students who do not attain the achievement criterion

(a) in a mastery learning situation (MMM)?

(b) in a non-mastery learning situation (NNN)?

Question II

What student characteristics can be used to predict mathematics achievement scores in a mastery learning setting?

Question III

What student characteristics can be used to predict mathematics achievement scores in a non-mastery learning setting?

Question IV

To what extent does the mastery learning method of teaching mathematics affect the following student characteristics:

(a) student attitude toward mathematics?

(b) student general self-concept of ability?

(c) importance that the student attaches to grades in general?

(d) student self-concept of mathematics ability?

(e) importance that the student attaches to mathematics grades?

A further question arises from a conclusion arrived at by Bloom (1973). He reported the unexpected emergence of a by-product of mastery learning which has been revealed in several of the studies conducted. There is evidence that students develop cooperation as opposed to competition in their learning. According to Bloom "a remarkable flowering of student cooperation" becomes apparent.

Another purpose of this study, therefore, was to conduct observations in the classroom to ascertain if cooperation rather than competition tended to emerge. Thus, the following question was raised:

Question V

Under mastery learning conditions, to what extent do students develop cooperation as opposed to competition in their learning?

DEFINITION OF TERMS

Mastery. The attainment of a pre-determined criterion.

Mastery learning. Learning defined in terms of mastery of specific sets of objectives.

Experience-oriented mastery learning. Mastery learning which involves a classroom setting in which the learner develops mathematical concepts by means of active participation.

Formative evaluation. A process of diagnostic testing which provides immediate and regular feedback to the student and the teacher regarding the student's progress during the course of instruction.

Unit. The entire mathematical content of the study.

Subunit. A subset of the unit comprising one-third of the content of the unit of mathematical content.

Basic learning task. A learning task whose mastery is fundamental to the mastery of a subunit.

Attitude. A predisposition to respond toward certain objects, conditions, or events either positively, negatively, or neutrally. Classroom attitudes are dimensions of motivation which, in turn, are directly related to the learning process (Jones, 1968).

Self-concept. A complex and dynamic system of beliefs which an individual holds true about himself, each belief with a corresponding value (Purkey, 1970).

Cooperation. The act of working together to one end.

Competition. The act of seeking or endeavouring to gain what another is endeavouring to gain at the same time.

Tutoring unit. A period of time of less than or equal to one minute during which a student was helped in his mathematical tasks by one of his peers, by the teacher or by a researcher.

Terms Relating to Carroll's Model of School Learning

Learning task. The learner's task of going from ignorance of some specified fact or concept to knowledge or understanding of it, or of proceeding from incapability of performing some specified act to capability of performing it.

Aptitude. The amount of time an individual student will need to learn a task under optimal learning conditions.

Opportunity. The time allowed for learning.

Quality of instruction. The degree to which the presentation, explanation, and ordering of elements of a task to be learned approach the optimum for a given learner.

Perseverance. The amount of time the learner is willing to spend actively engaged in learning.

Ability to understand instruction. The ability of the learner to understand the nature of the task he is to learn and the procedures he is to follow in the learning of the task.

THE NEED FOR THE STUDY

Although approximately two decades have passed, many educators still adhere to the assumption expressed by Conant (1959) that only a limited portion of American youth is capable of high level educational effort. Conant assumed that only 15 per cent to 25 per cent of high school students can profit by education in mathematics, science and foreign languages. Bloom et al. (1971) have expressed the opinion that teachers anticipate that, in round figures, one-third of their students will learn a great deal of what they serve to impart, one-third will adequately learn what is offered and the remaining one-third will either merely come to the mark or will fail.

It has been reported by Chauncey (1959) that the Russians do not share assumptions such as those of Conant. They assume that in the last three years of their educational program, with its heavy emphasis on mathematics, sciences and foreign languages, 99 per cent of their students will not only take the program but profit from it and complete it. Although Chauncey was unable to obtain complete data to determine how close the Russians came to achieving their expectation of 99 per cent success, he was able to estimate that "somewhere between 50 and 80 per cent of Russian students actually get through the ten-year school program".

It would appear that whatever is forecast will, in time, come to pass; the prophecy will, of itself, become fulfilled.

Of the subjects of the school curriculum, mathematics ranks very high in importance. Increasingly, the demands of today's technological world call for a high level of mathematical literacy among the members of society.

As technological advances are achieved, new trades and professions are created; at the same time, many existing trades and professions become obsolete. It has been estimated that, generally, each member of society may expect to be required to change his field of work several times during his working life. As a result of obsolescence, periods of retraining will be necessary to equip people for new occupations. Without a good understanding of the basic concepts of mathematics, the transition from one field to another may be painful or may not be possible. The transition may be unnecessarily protracted and even traumatic. Those who lack the basic skills to be retrained successfully will likely become unemployable. The loss to the individual and to society compels us to seek to prevent such situations from arising.

Consequently, a great need exists for all practitioners in the field of education to entertain seriously the proposition that almost all students can comprehend all that is offered. Block (1971) indicates the main benefits to be gained by such a belief put into practice. He contends that the practice of mastery learning:

1. enables 75 to 90 per cent of the students to achieve to the same high level as the top 25 per cent learning under typical group-based instructional methods;
2. makes student learning more efficient than conventional approaches. Students learn more material in less time.
3. produces markedly greater student interest in and attitude toward the subject learned than usual classroom methods.

A second need for this study is to demonstrate that a strategy can be developed whereby the teacher can manage the learning so that most of the students can learn most of what is taught.

A third need for the study is to examine the effect of certain student characteristics (e.g., intelligence, attitude, self-concept of ability) on student achievement. In turn, it was attempted to ascertain the effect of student achievement on these characteristics.

A fourth need for the study is to explore the phenomenon of increasing student cooperation as opposed to competition under mastery learning conditions.

Although many experiments have been conducted using mastery learning strategies, there is still a need to replicate these studies in various subject areas at all grade levels, using different instructional modes and in a wide range of environments. It is hoped that this study will contribute to the research on mastery learning strategies, particularly with regard to the characteristics of the students.

In summary, the need for the study is four-fold:

1. to demonstrate that most students can learn almost all of what is offered in school;
2. to devise a practicable strategy to achieve such a degree

of learning;

3. to examine the effect of certain student characteristics on student achievement and, in turn, to examine the effect of achievement on these characteristics;
4. to discover if student cooperation as opposed to competition develops under mastery learning conditions;

DELIMITATIONS OF THE STUDY

The study was delimited in the following ways:

1. only the 153 Grade 9 students and two teachers of one junior high school were involved;
2. the mathematical content was restricted to the study of polynomial expressions as prescribed by the Program of Studies of the Province of Alberta; and
3. experimentation in the school took place during a period of four weeks.

ASSUMPTIONS

The nature of the materials used and the design of the study, require that certain assumptions be made. These assumptions are as follows:

1. that the three subunits comprise equal amounts of work;
2. that the formative tests relating to each subunit are parallel;
3. that the items in the formative and summative tests are appropriate measures of the stated basic learning task objectives;

4. that the quality of instruction is consistent throughout the experiment;
5. that attitudes toward mathematics can be measured by a paper and pencil semantic differential instrument;
6. that self-concept of ability toward mathematics and other related affects can be measured by a paper and pencil multiple choice questionnaire;
7. that the cooperative behaviour of students observed during a few classroom visits represents an adequate sample of their usual behaviours under mastery and non-mastery conditions;
8. that all subjects interpret the questions in paper and pencil tests in the same way; and
9. that the presence of the two researchers influences the behaviours of the students of all classes to the same degree.

OUTLINE OF THE THESIS

This first chapter provided a brief introduction to the problem and the context in which it will be discussed, subsequently. Chapter II examines several of the more significant efforts in developing mastery learning strategies. From this examination a theoretical basis is established within which the results of the study are interpreted. The development of the materials used in the study and the rationale upon which they are based are described in Chapter III. Chapter IV gives details concerning the design of the experiment, procedures and the methods used in analysing the data. Chapter V contains the results of the study, the interpretations and the conclusions drawn. Chapter VI provides a summary of the study, discusses the results and implications, and indicates some areas where further research may usefully be undertaken. The concluding chapter, Chapter VII, describes some of the developments in mastery learning theory and practice which have taken place since this experiment occurred.

CHAPTER II

THE RELATED LITERATURE

INTRODUCTION

The first part of the review of the literature will focus on certain aspects of the literature on mastery learning with the purpose of identifying characteristics of mastery learning strategies and the theory of learning associated with them. In this way, a theoretical basis will be established within which the results of this study may be interpreted.

Several of the more significant efforts in the development of mastery learning strategies are discussed. In particular, the model of school learning propounded by Carroll, and Bloom's subsequent application of the model to a particular mastery learning strategy will be discussed in some detail

The second part of the review will be devoted to certain student characteristics and the effects which these characteristics have on the students' learning. In particular, the roles which attitude and self-concept play in learning will be discussed. Further, some factors which may affect attitude and self-concept will be considered.

Since this study sought to apply an experience-oriented approach to a mastery learning strategy, the third part of the review will focus on certain aspects of activity learning.

Finally, a brief section is included on the effects of a mastery learning approach on the cooperation or competition of students in their learning tasks.

MASTERY LEARNING

Although the development of effective mastery learning strategies has occurred only within the past ten years, the concept of learning for mastery has been with us for over half a century. Major attempts at producing mastery in students' learning took place in the 1920's. Perhaps the most worthy attempt of note is the Winnetka Plan devised by Carleton Washburne and his associates (1922) which grew from the dissatisfaction with the lock-step system. Under the lock-step system only a small proportion of students could proceed naturally at an arbitrarily established pace, thereby constraining the more capable and frustrating the weaker students.

The curriculum, under the Winnetka Plan, was divided into two parts; one containing the basic subjects (reading, arithmetic, language arts and social studies) which were divided into unit lessons and which each pupil had to master completely, and the other part containing group and creative activities (e.g., physical education and the fine arts) in which no specific standards had to be met. An underlying assumption was that every student, given enough time, could learn every concept. Thus, within the basic curriculum, time became the variable factor and achievement became the constant factor.

If each student had to master certain essential skills and reach certain goals at his own rate of progress, new evaluation processes were required. It was necessary to define the unit objectives to be mastered, to devise tests which would diagnose student weaknesses, and to provide self-correctional practice materials to enable the student to make good his deficiencies and to facilitate his progress to the next unit.

Washburne, Vogel and Gray (1926) reported on a comprehensive series of studies and experiments designed to evaluate the effectiveness of the Winnetka Plan. One of their findings indicated that the Plan enabled more students to keep up with the work appropriate to their age level. This was as a result of the student's proceeding at his own rate and not being required to repeat an entire grade in the event of difficulties in certain areas of the curriculum. The individualized program showed striking variability in students' rates of progress across subject matter areas. For example, in a comparative study in arithmetic achievement, it was shown that the fastest student was almost two years ahead of the average student, whereas the slowest student was roughly one year behind.

Although the Plan was a sincere effort towards "fitting schools to individuals" the system was, according to Carroll (1971), not a complete success, due probably, to the lack of an adequate technology of instruction prevailing at that time.

Another significant approach, developed by H. C. Morrison (1926) at the University of Chicago Laboratory School, had several features in common with the Winnetka Plan. However, whereas Washburne's concept of mastery dealt solely with cognitive objectives, Morrison's concept included affective and psychomotor objectives as well.

Block (1971) has summarized the major features common to both these approaches, namely:

1. Mastery was defined in terms of the particular educational objectives which each student had to achieve. These objectives were cognitive for Washburne; and cognitive, affective and psychomotor for Morrison.
2. Instruction was organized into clearly defined units comprising a set of learning materials systematically ordered to teach the desired unit objective(s).
3. Complete mastery was required before a student could proceed to the next unit. This was especially important in the Winnetka Plan where the units were sequenced so that the learning of each unit relied upon prior learning.
4. An ungraded diagnostic-progress test was administered at the end of each unit to provide feedback on the extent of the student's learning.
5. Wherever necessary, each student's original instruction was supplemented with learning correctives to enable him to complete his unit learning.
6. Time was used as a variable in individualizing instruction. Under the Winnetka Plan, student learning was self-paced and all the time necessary for mastery was permitted.

Due mainly to the lack of the requisite technology, interest in mastery learning waned so that by the late 1930's it had disappeared altogether. However, with the advent of programmed instruction a renewed interest in mastery learning occurred around 1960. Although programmed instruction served as a valuable aid in facilitating mastery for some students, it failed to provide a useful model of mastery learning.

Carroll's Model of School Learning

In 1963, John B. Carroll in his Model of School Learning provided a model for mastery learning which, in time, served to inspire Bloom to derive his concept of mastery learning. Bloom (1973) expressed the view that "in setting time as the central variable in school learning, Carroll produced a major shift in our thinking about education and educational research."

Carroll's model was based on the concept of a learning task which he defined as,

> The learner's task of going from ignorance of some specified fact or concept to knowledge or understanding of it, or of proceeding from incapability of performing some specified act to capability of performing it. (1963:723)

Bloom claimed that most of the goals of the school could be expressed in the form of learning tasks, or a series of such tasks. Since each task would be stated in testable form, the teacher would be able to determine when the task had been mastered to a satisfactory degree, i.e., when the criterion for satisfactory performance had been attained.

Carroll's model involves five elements, three of which are resident in the individual and two of which stem from external conditions, namely:

Factors residing in the individual.

1. Aptitude
2. Ability to understand instruction
3. Perseverance.

Factors in external conditions.

1. Opportunity

2. Quality of instruction.

These five factors may be combined in a formula which expresses the degree of learning for a certain individual working on a particular task. Thus,

$$\text{Degree of learning} = f\left(\frac{\text{time actually spent}}{\text{time needed}}\right)$$

The numerator of this ratio is a function of opportunity and perseverance, while the denominator is a function of aptitude, quality of instruction, and ability to understand instruction. That is,

$$\text{Degree of learning} = f\left(\frac{\text{1. Time allowed \quad 2. Perseverance}}{\begin{array}{l}\text{3. Aptitude \quad 4. Quality of instruction}\\ \text{5. Ability to understand instruction}\end{array}}\right)$$

Carroll claims that, regardless of the interests of the student or how he is motivated, if he spends the amount of time he needs on learning the task he will learn to criterion. Of course, the amount of time required will be short for the faster students and very long, perhaps infinitely long for the exceptionally slow. Whenever the quality of instruction and the student's ability to understand instruction are both optimal, the time required would be a minimum. On the other hand, if both these factors are less than optimal, the amount of time needed would be increased for the student.

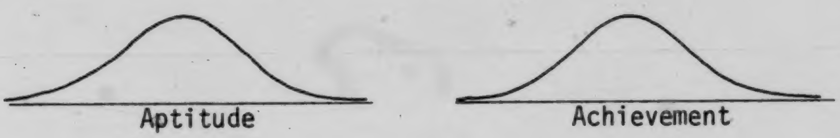
Bloom (1968) transformed Carroll's conceptual model into an effective working model of mastery learning. He reasoned that:

> If students are normally distributed with respect to *aptitude* for some subject and all students are given exactly the *same instruction* (the same in terms of amount and quality of instruction and learning time allowed) then achievement measured at the completion of the subject will be normally distributed. Under such conditions the correlation between aptitude measured at the beginning of the instruction and achievement measured at the end of instruction will be relatively high (typically about +.70). Conversely, if students are normally distributed with respect to aptitude

> but the kind and quality of instruction and learning time allowed are made appropriate to the characteristics and needs of each learner, the majority of students will achieve mastery of the subject. And, the correlation between aptitude measured at the beginning of instruction and achievement measured at the end of instruction should approach zero.

Figure 1 provides a representation of these two situations.

Uniform Instruction Per Learner



Optimal Instruction Per Learner

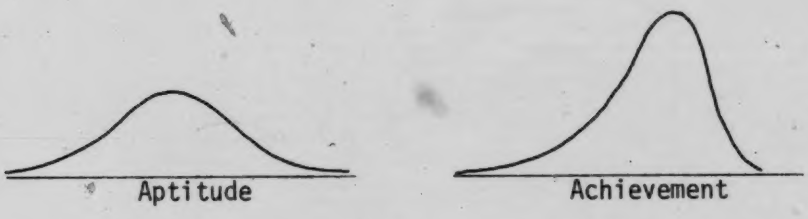


FIGURE 1

DISTRIBUTION OF APTITUDE AND ACHIEVEMENT
FOR UNIFORM AND OPTIMAL INSTRUCTION

Block (1971) describes the strategy which Bloom proposed to implement these ideas for use in the classroom where the time available for instruction is relatively fixed. The major features of this strategy were:

1. Mastery was defined in terms of a specific set of major objectives (content and cognitive behaviour) which the

student was expected to exhibit by a subject's conclusion.

2. The subject was subdivided into a number of smaller learning units (e.g., two weeks' instruction) and their objectives defined.
3. Simple feedback/corrective procedures were used to ensure that the student's unit instruction was of optimal quality and that the individual's problems were overcome before the student proceeded to the next portion of the work.

This approach represented a substantial improvement over previous strategies in two significant ways. First, the work of Gagne (1968) and Bloom et al. (1956) resulted in greatly improved feedback instruments. By describing more precisely the structure of the learning units in terms of their components and the interrelationships among these elements, a blueprint was thus available for the diagnostic instruments. Bloom's concept of the role of formative evaluation as an integral part of the teaching-learning process and as a means of continuous feedback facilitated constant modification of the process of bringing about mastery. Second, Bloom's approach made use of a wider variety of instructional correctives (e.g., small-group study sessions, individualized tutoring, alternative learning materials, and reteaching).

Although the success or failure of mastery learning work depends greatly on the effectiveness of the formative tests in zeroing in on the learning needs of the pupil, the real key to success resides in the degree to which the student is motivated and aided in correcting his learning difficulties at the appropriate stages in the learning process.

Block (1971) has developed operating procedures for mastery learning. The major features of these procedures are:

1. the acceptance of the view that almost all students can learn to high levels is absolutely essential to the development of any effective strategy;
2. the selection of subjects which,
   - require either minimal prior learning or previous learning which most learners already possess,
   - are sequentially learned,
   - tend to be closed and emphasize convergent rather than divergent thinking;
3. the objectives of instruction are specified in terms of the skills to be learned. A crucial step is the translation of these objectives into specific summative evaluation procedures to grade and judge the student's learning at the completion of the study of the subject;
4. the establishment of an absolute performance standard and the rejection of normal-curve grading;
5. the establishment of formative evaluation procedures;
6. the production of a wide variety of learning correctives to supplement original instruction;
7. the determination of the frequency of the feedback/correction procedures;
8. the allocation of learning-instruction time; and
9. the student orientation sessions.

SELF-CONCEPT AND SCHOOL ACHIEVEMENT

Purkey (1970) has formed a composite definition of the self from various definitions of the self given by Lecky (1945), Rogers (1951), Jersild (1952), and Combs and Snygg (1959). Thus, Purkey defines the self as "a complex and dynamic system of beliefs which an individual holds true about himself, each belief with a corresponding value." He identifies some of the most important characteristics of the self as follows:

1. the self is organised and dynamic;
2. that to the experiencing individual the self is the centre of his personal universe;
3. that everything is observed, interpreted, and comprehended from this personal vantage point; and
4. that human motivation is a product of the universal striving to maintain, protect and enhance the self.

Purkey believes that there are times when the self-image appears to shift abruptly - as on a child's first day of school, upon graduation, marriage, and retirement - but generally the self is ultraconservative. Lecky (1945) has reported that the self resists change and strives for consistency. Engel (1959) who investigated the stability of the self over a two-year period of adolescence found that the self resists modification to a surprising extent. Nevertheless, Purkey maintains that the self will change if conditions are favourable. He states,

> If the child sees the educative process as meaningful and self-enhancing, and if the degree of threat provided by the school experience is not overpowering, then he is likely to grow in self esteem and in academic achievement, Very few students want to be failures at learning, just as very few teachers want to be failures at teaching.

Students are in attendance at school during the most important periods of development of their physical, psychological and social characters. During a period of approximately 12 years they encounter a curriculum which is both explicit and implicit. The explicit curriculum consists of the subject matter (e.g., mathematics, science, physical education) and mastery in these subject areas can be measured in many ways. By means of the implicit curriculum the student is made aware of his position in the academic and social environments of the school. Although it is more difficult to observe and evaluate, it is the implicit curriculum which has the greater effect upon the student's perception of his 'self'. Many authors, e.g., Irwin (1967), Brookover (1964) and Purkey (1970) have stated that it is the student's self-concept which determines his performance in school. Thus, the explicit-implicit nature of the curriculum affects the development of the student, physically, psychologically and socially.

According to Bloom (1971), subject mastery affects the self-concept of the student. Everyone seeks positive recognition of his worth and will perceive himself as adequate in those areas where he is given assurance of his success or competence. Consequently, a student must be given many opportunities for rewards. Bloom states:

> Mastery and its public recognition provide the necessary reassurance and reinforcement. . . . Mastery learning can give zest to school learning and thus help develop a life-long interest in learning. It is this continual learning that should be the major goal of modern education.

When the student entering school attempts to complete his first learning task he receives some form of judgment from his teachers and his peers. This judgment, in turn, affects the way in which he will approach and attempt to perform subsequent tasks. Bloom (1974) and

Payne (1975) believe that, as the student is exposed to the continual assessments of his teachers, his peers and himself, his self-concept of ability develops. The cyclical process of task, attempt and judgment, repeated on numerous occasions throughout each day, results in the development of a consistency of performance in which the student begins to align his past record of achievement with his prevailing feelings of competency in the situation.

The interest of the student with regard to the subjects of the curriculum will vary as a function of his developing self-concept; his performance, subsequently, will be affected as his self-concept continues to evolve. It has been demonstrated by Anderson (1952) that a student who is convinced of his inability to perform a task will seek to avoid it by some means or another. Among the techniques employed are: attempting to disguise lack of ability; making no effort to complete the task; or refusing to acknowledge the importance of the activity. Gever (1970), Sebeson (1974) and Black (1974) have given proof of the unwillingness of students to enter into activities which have previously resulted in negative feedback. Andrews (1971) and Beare (1975) have shown that consistent negative feedback will produce a correspondingly negative self-concept, particularly so in an academic setting.

Torshen (1969) has summarized the studies which have shown a relationship between self-concept and school achievement. She found a correlation of approximately 0.50 between academic self-concept and academic achievement. Although Torshen focussed mainly on academic self-concept of the student, Quandt (1972) and others have sought to show the many facets of the self-concept. He has expressed opinions regarding the role of the parent in enhancing the student's self-concept by being realistic in his academic expectations and judgments. Further, it may be possible for a student to have a high social or physical self-concept while, at the same time, to have a low academic self-concept. The numerous facets of the complex self-concept serve to determine the form of the motivation of the student toward various activities. In fact, the level at which a student is functioning may be determined by his self-concept and not by his potential.

Ample evidence exists to prove that intelligence alone does not account for the level of achievement in students.

One of the most extensive studies in the area of self-concept and achievement was conducted by Brookover, Patterson and Thomas (1964). The study was based on the view that a person's self-concept is developed by means of his interactions with persons who are significant to him and that these interactions, in turn, influence his future behaviour. The study focussed on one aspect of the student role (academic achievement) and one aspect of self-concept (self-concept of academic ability) and sought to determine how these two aspects were interrelated. Two forms of a Self-Concept of Ability Scale were administered to a sample of 1,050 Grade 7 students (equal numbers of male and female students). One form measured general self-concept and the other measured self-concept in each of four specific school subject areas; arithmetic, English, social studies and science. Grade point average was used as a measure of academic achievement in each of the subject areas and intelligence was controlled.

The study produced evidence to establish the following interrelationships:

1. General self-concept and academic performance were posi-

tively and significantly related. (+.57 for males and +.57 for females): the relationship was maintained even with I.Q. controlled.

2. There were specific self-concepts of ability related to specific areas of academic performance. These specific self-concepts were significantly better predictors of specific subject achievement than was general self-concept.
3. A positive and significant relationship was found between general self-concept and the student's perception of how a few significant persons evaluated him. His self-concepts in the various subjects were related to his perception of how a number of other persons evaluated him as a student.

Brookover et al. (1964) expressed the opinion that self-concept is a key factor in role performance and that changes in self-concept should bring about changes in performance.

Block (1971) considers these findings important for several reasons, namely:

1. the possibility that self-concept may be changed;
2. if changes in self-concept may lead to changes in academic performance, then it is also possible that changes in academic performance may lead to changes in self-concept. The idea that there are general and specific self-concepts of ability suggests that we might change self-concept in specific subject areas by increasing a student's performance without necessarily first changing his general academic self-concept. If enough changes were realized in his specific self-concepts this perhaps would lead eventually

to changes in his general self-concept;

3. it is clearly shown that self-concept derives from the individual's perception of the evaluations that personally significant people make of him.

Torshen (1968), in her study with approximately 100 Grade 5 students of lower and middle socio-economic classes selected from three school districts, found a significant, positive relationship between the teachers' evaluations of the students' achievements and the self-concepts of the students. The removal of the influence of achievement test performance does not significantly reduce the relationship. However, if the influence of the teachers' evaluations is removed, the relationship between performance in achievement test and students' self-concept is not significant. Torshen's findings, therefore, support the proposition that teachers' evaluations of students have a greater influence upon students' self-concepts than do their objective achievement tests evaluations. This may be attributed to the fact that students receive feedback regarding their achievement more frequently from their teachers than they do from occasionally administered objective achievement tests. Students may regard teachers' evaluations as being more relevant assessments of their academic competence. In this way teachers' evaluations may provide the basis for students' concepts of their academic ability.

Modu (1969) in his study involving over 2,000 college and university students hypothesized that subjective feelings of failure or success cause personality changes in students and provide specific cues for positive or negative self-evaluation. Changes in a number of affective variables (e.g., self-rating level of aspiration, life goals and interpersonal achievement) were found to be related to the students' grade discrepancies. Among the more noticeable changes was that of self-rating which thus stresses the importance of self-esteem as a sensitive barometer of perceived cognitive changes. Since self-esteem is responsive to subjective feelings of success or failure, it may be changed by an appropriate manipulation of cognitive achievement, even in late adolescence.

Modu thus demonstrated the need to re-examine existing grading practices and to reflect on the effects that a particular mark has on the student's self-evaluation. Further, it is suggested that learning strategies which promote higher levels of achievement and changes in grading procedures may prevent losses in a student's self-esteem, thus helping to prevent severe emotional disturbance.

Sears (1940), in her study with Grades 4, 5, and 6 students, confirmed the hypothesis that experiences of success and failure influence levels of aspiration. Feather (1966) indicated that prior success or failure had an important influence on the expectations of an individual and on his actual performance. Changes in expectations of success were greater following uniform initial failure than uniform success. In certain cases a student's general orientation toward a task (success or failure oriented) appeared to exaggerate further the influence of the initial success or failure condition on his expectations of success.

Bloom (1971) in discussing the affective consequences of school achievement sums up the matter:

> . . . successful experiences in school are no guarantee of a generally positive self-concept but they increase the probability that such will be the case. In contrast,

> unsuccessful experiences in school guarantee that the individual will develop a negative academic self-concept and increase the probability that he will have a generally negative self-concept. But the individual strives desperately to secure some assurance of his self-worth; if he is denied it in one area, he will search for it elsewhere. The likelihood of his finding it is considerably decreased by consistent lack of success in school.

ATTITUDE

In this portion of the literature review reference will be made to some of the more significant work on attitude. Although the relationship between attitude and school achievement in general will be examined, particular regard will be paid to the relationship between student attitude and achievement in mathematics.

The literature contains many definitions of attitude. One definition which has been generally accepted is that of Jones (1968). He states that attitude is,

> A predisposition to respond towards certain objects, conditions or events either positively, negatively or neutrally. Classroom attitudes are dimensions of motivation which in turn are directly related to the learning process.

Wilson (1961) has stated that every affective construct has an object associated with it. People do not just have attitudes, they have attitudes toward something. He believes that an attitude may be considered to have three components, namely:

1. the object which may be a school subject, e.g., mathematics;
2. a feeling, which has a direction or balance and some strength; and
3. a tendency to act on the object according to the balance and strength of the feeling.

According to Chein (1948) the learning processes in which a child engages play a major role in the development of his attitudes. Johnson (1957) is of the opinion that learning involves emotional vectors such as attitudes. The attitudes which the student develops will likely stimulate him toward or prevent him from the further study of mathematics. If a student is to enjoy mathematics he must engage in activities which appear significant and in which he encounters success.

The proponents of the Nuffield Project contend that attitudes toward mathematics may be formed as early as the primary school. Aiken (1970) states that the junior high school years are crucial in that this is a period when student attitudes toward arithmetic reach peak development.

Odynski (1972) has reported on the problem of teaching low achievers in high school. In the case of mathematics, the problem is compounded by the dislike which many students have for the subject. He reports that students who display extremely negative attitudes toward mathematics often cease their study of mathematics at the earliest opportunity which is usually upon their receiving credit in one Grade 10 mathematics course (the minimum prerequisite for the high school diploma).

Rhine (1958) has commented upon the intense desire of many students to avoid mathematics. He believes that there is nothing about mathematics itself that could cause people to dislike it. Nevertheless, if the subject of mathematics possesses no intrinsic characteristics which could engender such antipathy, why do so many students seek to avoid it? Gough (1954) has attributed this "mathemaphobia", fear of and dislike for the subject, to unhappy previous experiences. When the student experiences frustration instead of satisfaction, feelings in the form of dislike toward mathematics become established.

Poffenberger and Norton (1959) studied the effect which parents have on the student's attitude and performance. They found that parental encouragement for students to take mathematics courses in high school was significantly related to student attitudes toward mathematics. Further, student attitudes and performance in arithmetic and mathematics were affected by the teacher from Grade 1 through high school. They state that the student:

> ...carries into his high school mathematics attitudes that are long in building and difficult to change. The fact that so many high school students have negative attitudes toward mathematics makes the job of the teacher doubly difficult and indicates the need for outstanding teachers in terms of knowledge of subject matter, teaching ability, personality, and understanding of adolescents. (Poffenberger and Norton, 1959, p. 175)

Several researchers, including Aiken and Dreger (1957), have demonstrated that the attitudes of students are related to their experiences with their former mathematics teachers. Significant relationships have been shown to exist between:

1. the mathematics background of the teacher and student achievement in algebra;
2. the teacher's attitude toward algebra and the students' attitude toward the subject; and
3. the judgment of the teacher regarding the practical value of algebra and the judgments of the students.

Several studies by Young (1932), Dutton (1968), Chase (1949) and Dutton and Blum (1968) cite several reasons for loss of student interest in a subject. These reasons are as follows:

1. the failure of the student to perceive a need for the course;
2. the uninteresting material presented;
3. the monotonous methods employed; and
4. the difficulty encountered by the student.

Dutton also observed that liking arithmetic influences the volume of work attempted, the effort expended and the learning acquired. He contends that although lasting attitudes toward arithmetic develop through each of the grades, the developments at Grade 5 and Grade 7 are of particular significance.

Bloom (1971) has rather succinctly stated the causes of negative attitudes and their consequences:

> Attitude generalizes to the whole institution of the school, to most of the school subjects, to the staff of the school, and even to the students who attend the school. In effect, repeated evidence of inadequacy in school makes the entire institution the source of the individual's sense of inadequacy and he must avoid the institution or find some way of reducing the amount of pain it gives to him. This he does by efforts of retreating, attacking, or minimizing the school's effects on him. Such negative attitudes, if developed fully enough, may have consequences for all later efforts to do school learning or learning in any way related to schools.

Efforts must be put forth to eliminate as far as possible the negative attitudes of students. Attitudes, although resistent to change are not permanent; they may be changed. Kurtz and Swenson (1951) and others believe not only that attitudes affect achievement but also that achievement affects attitudes. Kurtz and Swenson found that attitudes toward educational achievement, the school, and the importance of an education were more closely related to the achievement scores of the students than to their ability scores. High attitude scores and high achievement were related as were low attitude scores and low achievement. Consequently, teachers must devise ways of ensuring greater levels of achievement for large numbers of their students. How may this be done?

ACTIVITY-ORIENTED APPROACH

In the belief that the "traditional" method of teaching school subjects is not conducive to improving student attitudes and achievement, researchers have experimented with other teaching approaches, for example, the activity-oriented approach.

Kieren (1969) has defined activity learning as ". . . school learning settings in which the learner develops mathematical concepts through active participation." One form of activity learning involves the manipulation of physical devices, the use of games, or taking part in experiments with physical objects. Another form of activity learning requires the student to engage actively in the process of forming mathematical ideas for himself, during which he may or may not use any physical devices.

Vance (1969) attempted to determine the effects of a mathematics laboratory approach on the achievement and attitude of Grades 7 and 8 students. He found that the experimental group experienced no adverse effects in achievement. However, although the resulting differences in attitude measures were not significant, they favoured the group which manipulated the concrete materials. Tests of immediate learning, cumulative achievement, higher level thinking and problem solving, and divergent thinking indicated that the students in the two types of experimental classes benefited mathematically.

Swick (1960) tried to discover the effect that the use of multisensory concrete teaching aids would have on student achievement, student attitude and teacher attitude. The results of the study showed an improvement in the attitude of the Grade 2 and Grade 3 students. Teacher attitudes toward arithmetic and the continued use of the multisensory teaching aids were also indicated.

Plank (1950) used Montessori materials in her work with students ranging in ability from retarded to high level of ability. She found that the students showed a long span of interest and greater perseverance than could be expected of children of their age group (five to twelve years of age). The children of the retarded and accelerated groups found the materials equally interesting. They became relaxed and their thinking was stimulated.

Comprehensive reviews of the literature on activity learning have been done by Kieren (1969) and Kieren and Vance (1971). Although results have not always been conclusive, there is some evidence to support the contention that students of low ability achieve higher in activity situations while other students learn at approximately the same rate as those learning under regular classroom conditions.

Others, such as Kidd, Myers and Cilley (1970), Rouse (1972) and Reys and Post (1973) have demonstrated the effectiveness of activity approaches in the learning of mathematics. Biggs and MacLean (1969) stress two aspects of activity learning:

1. the student must be permitted to do things over and over again; and
2. he should find the practice enjoyable.

These two aspects serve to ensure that one of the crucial requirements for success in a mastery learning situation, the willingness of the student to persevere, will be met.

Dienes (1960, 1964) stresses the need for multiple embodiment, that is, the student's need to explore several physical manifestations of a concept and to synthesize these experiences in forming the concept.

The concept of an activity-oriented approach to learning of mathematics is therefore well supported.

The definition of activity-oriented mastery learning used in this study, namely, "Mastery learning which involves a classroom setting in which the learner develops mathematical concepts by means of active participation" has been derived from Kieren's definition. The particular form of activity learning used in this study does not require the use of manipulative devices.

COOPERATION VERSUS COMPETITION

Bloom (1973) has indicated that one of the by-products of mastery learning which emerged quite unexpectedly is the degree to which students develop cooperation as opposed to competition in their learning. Under mastery learning conditions, criteria for mastery are set in absolute terms; the proportion of students attaining mastery is not fixed but is determined by achievement. Thus, pupils soon recognize that they are no longer in competition with one another for scarce rewards. In conditions where students are encouraged to work together to help one another with specific elements of learning identified by formative tests, they perceive their counterparts as a source of instructional aid in overcoming specific learning difficulties. As a result, "a remarkable flowering of student cooperation" becomes apparent.

As Bloom points out, it is as yet too soon to ascertain whether these new attitudes and relationships persist after the completion of the course. It is his belief that,

> . . . student perception of alternative learning resources beyond the teacher and textbook represents a by-product that may be more important in the long run than the actual benefits derived from the increased achievement in a particular subject.

CHAPTER III

MATERIALS PREPARED FOR THE STUDY

INTRODUCTION

A major aspect of this study was the production of materials which would enable the teachers to manage student learning by means of a strategy which adhered to the fundamentals of mastery learning theory. Block (1971) enunciated the fundamentals of any mastery learning strategy as:

1. Mastery is defined in terms of the particular educational objectives which each student has to achieve.
2. Instruction is organized into clearly defined units comprising a set of learning materials systematically ordered.
3. Complete mastery is required before a student proceeds to the next unit.
4. Ungraded diagnostic-progress tests are administered at the end of each unit to provide feedback on the extent of the student's learning.
5. Original instruction is supplemented with learning correctives wherever necessary.
6. Time is used as a variable in individualizing instruction.

Efforts were made to adhere closely to these fundamentals in the preparation of the materials.

MATHEMATICAL CONTENT

It has been stated (Block, 1971) that the most successful mastery learning techniques have utilized content which:

1. required either minimal prior learning or learning which most of the students had acquired;
2. was learned sequentially; and
3. was closed and emphasized convergent thinking rather than divergent thinking.

The content which was selected, namely, that of finding the products of algebraic polynomials together with the factoring of such polynomials appeared to possess the above characteristics. Further, the topic selected was conducive to experience-oriented modes of instruction which was also a requirement.

The content used is outlined in unit III, section E, subsections 3 and 4. (Appendix A)

BASIC LEARNING TASK OBJECTIVES

The total content selected (the unit) was divided into three subunits. Efforts were made to produce subunits which were of equal length, of equal difficulty, and which could be mastered by most students within six periods of instruction each of approximately 40 minutes duration. Each of these three subunits was divided into 10 basic learning tasks of equal length and difficulty. The basic learning tasks were ordered hierarchically so that each task required mastery of the previous task. Each basic learning task was presented in the form of an objective which comprised a clear and simple statement of educational intent, two sample test questions and their complete solutions. These objectives are contained in Appendix B. The statements of objectives served as the basis for the worksheets, formative tests, experiences, review sheets, summative and postsummative tests prepared for this study.

MATERIALS PREPARED

Worksheets

The worksheets were designed to provide practice for the student during the interval between the initial instruction and the administration of the first formative test for each subunit. The items followed the same order and were parallel forms of the sample questions on the objective sheets. Appendix C contains a sample worksheet.

Formative Tests

Crucial to the success of mastery learning methods are the effectiveness and efficiency of the formative tests. The purpose of formative evaluation has been stated by Airasian (1971) and Block (1971) to be that of guidance of the teaching-learning process in which immediate and continuous feedback on a student's progress is received during the course of instruction.

Four formative tests of parallel form were prepared for each subunit. The items were parallel forms of the corresponding examples of the objective sheets. Each formative test contained two items on each objective of the relevant subunit. The tests were designed such that the majority of students could complete them within 20 minutes. Since the tests were easily and quickly marked, immediate feedback was obtained. If a student did not master an objective he was directed to an experience designed to assist him in overcoming his difficulty.

The tests served solely to diagnose difficulties. No scores were calculated nor grades assigned.

Appendix C contains the formative tests for Subunit III.

Experiences

According to Block (1971) the sole function of the correctives in a mastery learning situation is to supply the student with the instructional cues and the active participation and practice and the amount and type of reinforcements which he needs to complete his unit learning.

Since the original instruction for each subunit was carried on during a period of one to one and one-half class periods it was anticipated that most students would require correctives for several basic learning tasks. It was also anticipated that some students would require more than one corrective in overcoming difficulties experienced with a particular basic learning task.

Three series of correctives (experiences), A, B and C were prepared for each of the basic learning tasks. The rationale underlying the construction of the experiences was as follows:

1. The A-experiences would be used only by those students who did not display mastery in the first formative test. These experiences would provide the student with more practice since many of his difficulties would likely be due to the lack of practice.
2. The B-experiences would be used by students who still had

not mastered objectives as evinced by the second formative test. These experiences would provide the student with a more detailed explanation of the steps involved in mastering a task. Complete solutions would generally be demonstrated. If a student was unable to understand these solutions he would be encouraged to ask for help.

3. The C-experiences would be used by students whose formative tests indicated considerable difficulty in mastering the tasks. These experiences would explain the tasks in a basic step by step fashion. Occasionally, short cuts or mnemonics would be utilized in an effort to attain mastery within the time available.

It was intended that the students could complete these experiences by themselves within five minutes, perhaps by referring to the worked examples on the objective sheets. The answers to the questions were always available in some form on the experience sheets so that the student could monitor his progress immediately. Efforts were made to make the three levels of experiences interesting by varying the format to include anagrams, magic squares, stories, dialogues and tables.

A complete set of experiences for subunit III is contained in Appendix C.

Review Sheets

A review sheet which was prepared for each subunit contained two questions on each of the subunit's objectives. The questions were of parallel form to the examples in the objective sheets. The review sheet was distributed at the end of each subunit. For the student who had attained early mastery on some or all of the objectives the review sheet served to ascertain if he could still attain criterion on those learning tasks. The expectation was that he would work to correct any deficiencies indicated by the review sheet. For the student who was experiencing difficulty the review sheet provided additional practice.

Appendix C contains an example of a review sheet.

Summative and Postsummative Tests

The summative test was used for determining the achievement of the student on the 30 basic learning tasks of the unit. This test was administered at the end of the third subunit of instruction. The postsummative test performed the same function and was administered two weeks later.

Both tests were parallel in form and contained one question on each learning task.

Item analyses showed that the Pearson product-moment correlation on the two tests was 0.86. Table I indicates the mean, variance and Kuder - Richardson 20 Reliability scores of both tests.

TABLE I

MEANS, VARIANCES, AND KUDER - RICHARDSON 20 RELIABILITY SCORES FOR SUMMATIVE AND POSTSUMMATIVE TESTS

TEST	N	MAXIMUM SCORE	MEAN	VARIANCE	K-R 20 REL
Summative	152	30	18.2	75.9	0.92
Postsummative	153	30	20.4	69.1	0.92

Appendix C contains copies of the summative and postsummative tests.

Attitude Toward Mathematics Test

The instrument used to measure attitudes toward mathematics is that developed by Odynski and others (Odynski, 1972). This test employs a semantic differential type of scale between pairs of antonyms. The weighing of choices ranges from five (the most favourable choice) to one (the least favourable choice). The sum of the weights of a student's choices constitutes his score on the scale. This attitude scale was originally devised for students enrolled in the Grade 10 mathematics course for students of low ability (Mathematics 15). Considerable care was exercised to employ terminology which was comprehensible to such students.

A test-retest procedure was used by Odynski to establish the reliability of the attitude instrument. The reliability of each of the 25 word pairs on the attitude test was established by comparing each student's two responses to each word pair. The reliability for each word pair was calculated by ascertaining the number of student responses on the posttest that were within plus one or minus one on the pretest and dividing this number by the total number of students responding to the word pair. The reliabilities of the word pairs ranged from 0.73 to 1.00 with the mean of the 25 word pairs being 0.89.

On the grounds of comprehensibility of the terminology used and the reliability, this attitude scale was deemed to be suitable for use with the subjects of this study.

Appendix E contains the Attitude Test.

Self-Concept of Ability Test

The instrument used was adapted from the one developed by Brookover and others (1962) in the course of their study "Self-Concept of Ability and School Achievement." Although numerous instruments have been developed to measure many aspects of self-concept and self-ideal, few seem to have focused adequately on the concept of academic ability. The Brookover instrument serves to fill this void. Although the test was designed originally for Grade 7 students it was deemed to be appropriate for the Grade 9 students of this study.

The reliability of the Self-Concept of Ability scales determined by Hoyt's method was 0.82 for boys and 0.77 for girls. A cross validation of the General Self-Concept scales prediction of grade point average (GPA) indicated that the predicted GPA correlated with actual GPA 0.70 for girls and 0.71 for boys.

The instrument used in this study was adapted from Brookover's original one and consisted of the following tests:

1. General Self-Concept of Ability. This test consists of eight five-choice items. The items are scored from five to one with the higher self-concept alternatives receiving higher values. The total score was used in the analyses.
2. General Importance of Grades. This test consists of seven four-choice items and one five-choice item. The items were coded from four to one, or five to one with the alternatives indicating greater importance of grades assigned the higher values. The total score was used in the analyses. Subsequent investigations of the importance of grades (IG) scale has indicated that the items are not unidimensional

but are composed of two dimensions only moderately correlated. The two dimensions have been designated as an absolute importance of grades (AIG) and competitive importance of grades (CIG). The former is concerned with doing well in an absolute sense while the latter is concerned with the importance of doing better than others in a competitive situation.

3. Self-Concept of Ability in Mathematics. The items for these scales are directly parallel to the general self-concept of ability items except that they relate specifically to mathematics. The eight-item tests are scored like the general self-concept test.
4. Importance of Mathematics Grades. These items are analogous to the General Importance of Grades test items except that they refer specifically to mathematics and there is one fewer item. The test is scored in the same way as the General Importance of Grades test.

Appendix E contains the Self-Concept of Ability Test.

CHAPTER IV

THE EXPERIMENTAL DESIGN

THE SAMPLE

The Grade 9 students of Sir George Simpson Junior High School, St. Albert, Alberta, participated in the study. These students were selected for the following reasons:

1. The number of students enrolled in Grade 9 was large (153 students).
2. The six classes of students were quite heterogeneous. With the exception of a few problem students the assignment of students to classes was completely random.
3. Only two teachers were responsible for the teaching of mathematics to the Grade 9 classes. Both teachers were willing to participate and were highly recommended to the researchers.
4. The mathematics program contained an area of content which was conducive to a mastery learning strategy.

ASSIGNMENT OF CLASSES TO TREATMENTS

The school operated on a six-day cycle, so the content was divided into three parts (subunits) each of which could be dealt with during six class periods. Four different treatments were decided upon:

1. mastery treatment for all subunits (MMM);

2. non-mastery treatment for all subunits (NNN);
3. non-mastery treatment for the first subunit and mastery treatment for the other two subunits (NMM); and
4. non-mastery treatment for the first two subunits and mastery treatment for the last subunit (NNM).

Attempts were made to minimize the effects of differences in the quality of instruction by randomly assigning one MMM treatment, one NNN treatment and either one NMM or NNM treatment to the three classes taught by each teacher. Table II illustrates the design.

TABLE II

TREATMENTS ASSIGNED TO EACH CLASS

CLASS SUBUNIT	9B	9F	9A	9D	9C	9E
I	M	M	N	N	N	N
II	M	M	M	N	N	N
III	M	M	M	M	N	N
TEACHER	X	Y	X	Y	X	Y

At least one researcher was present at all times during each mastery or non-mastery class period. They gave assistance to any student who requested it or seemed to require help.

DESCRIPTION OF MASTERY TREATMENT

On the first day of the experiment, students were apprised of the fundamental aspects of the treatments (e.g., the functions of the formative, summative and postsummative tests). During the remainder of this first period and for part of the next period the teacher dealt with each of the 10 objectives of subunit I in turn. Worksheets were distributed at the end of the first period to afford the students the opportunity of further practice.

When the teacher had concluded his instruction of the 10 objectives, the first formative test was administered. The formative tests were returned to the students at the beginning of the third period. Each student was thus informed of the tasks which he had mastered and supplied with A-experiences to help him succeed with the tasks which he had been unable to master.

Although the experiences were designed to enable the students to work independently, they were permitted, and frequently encouraged, to seek help from their fellow students, the teacher or the researchers.

Whenever a student displayed mastery on all 10 objectives he was advised to participate in the several interesting and recreational activities in the "activity corner." These activities were designed such that they did not interact with the content of the unit.

As soon as a student had completed the A-experiences he wrote the second formative test (test A). Wherever non-mastery was indicated, B-experiences were supplied. Formative test B was written when the student felt he had mastered the objectives. If necessary, C-experiences would be given and formative test C subsequently written. Thus, a student could work through three levels of experiences to help him master each learning task.

At the beginning of the final period of the subunit (period six) the students who had attained mastery worked through the review sheet. The students who were still working on experiences at the end of the

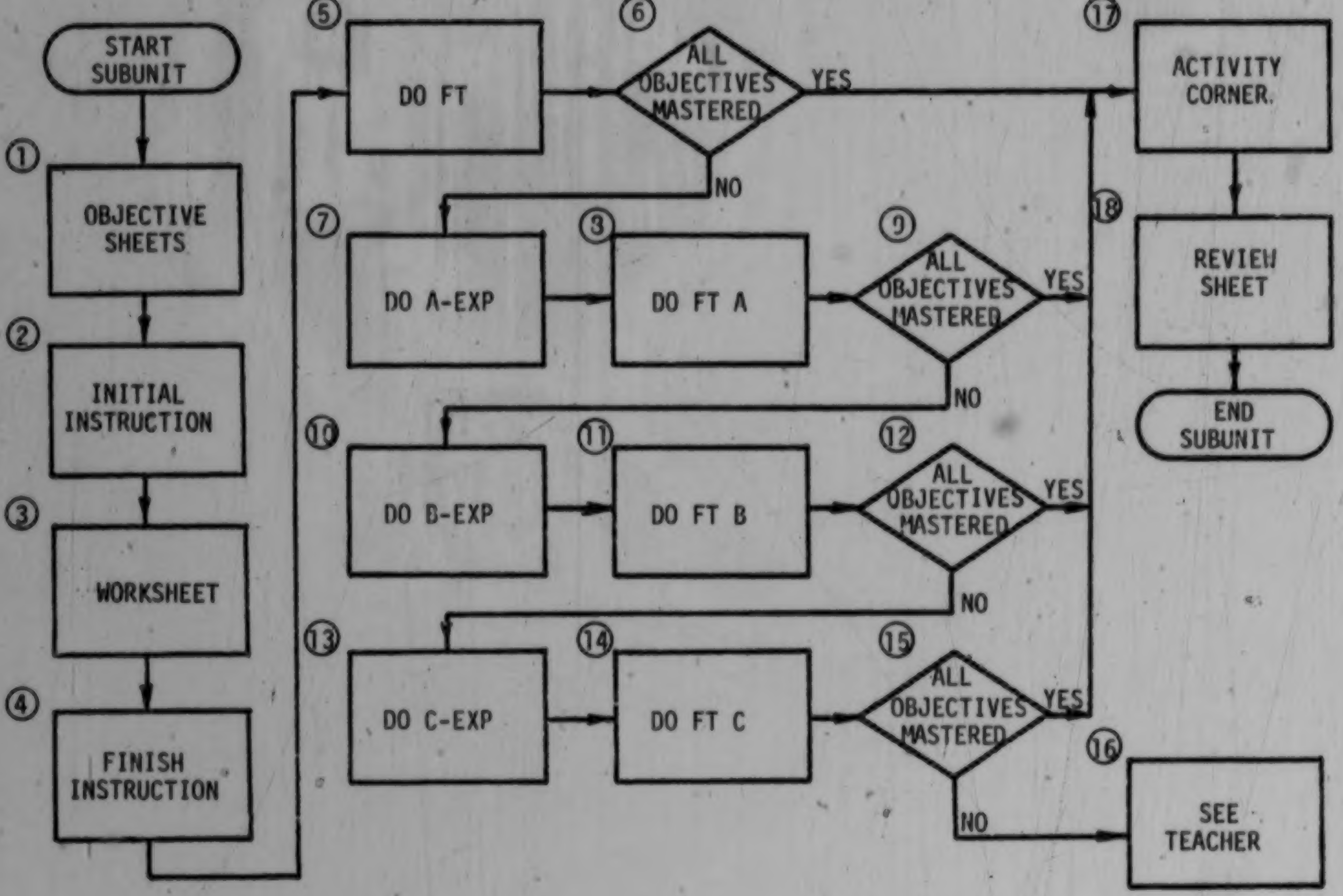

START SUBUNIT
①
OBJECTIVE SHEETS
②
INITIAL INSTRUCTION
③
WORKSHEET
④
FINISH INSTRUCTION
⑤
DO FT
⑥
ALL OBJECTIVES MASTERED
YES
NO
⑦
DO A-EXP
③
DO FT A
⑨
ALL OBJECTIVES MASTERED
YES
NO
⑩
DO B-EXP
⑪
DO FT B
⑫
ALL OBJECTIVES MASTERED
YES
NO
⑬
DO C-EXP
⑭
DO FT C
⑮
ALL OBJECTIVES MASTERED
YES
NO
⑯
SEE TEACHER
⑰
ACTIVITY CORNER.
⑱
REVIEW SHEET
END SUBUNIT

FIGURE 2: FLOWCHART FOR ONE SUBUNIT OF INSTRUCTION USING THE MASTERY TREATMENT

period were expected to do the exercises in their own time.

Similar procedures were used for subunits II and III.

Figure 2 illustrates, by means of a flowchart, the various stages in the completion of a subunit by the mastery treatment students.

DESCRIPTION OF NON-MASTERY TREATMENT

The non-mastery treatment dealt with the same mathematical content as the mastery treatment. Also, the amount of time devoted to the content was the same as for the mastery treatment. The sole difference lay in the kind of instruction.

At the beginning of the first period the objective sheets of subunit I were distributed. The teacher then followed his usual practices of presenting one or more objectives, doing examples on the blackboard with explanations as required. Students worked through appropriate sections of teacher-prepared worksheets. Their answers were checked before the teacher moved on to the next objective(s). This instructional mode was followed during periods two, three, four and five.

The teacher spent some time at the beginning of period six in answering student questions regarding the material covered. The students then wrote a test whose questions were identical to the first formative test. The marks obtained were used in determining the grade to be given to each student.

Similar procedures were used for subunits II and III.

Figure 3 illustrates, by means of a flowchart, the various stages in the completion of a subunit by the non-mastery treatment students.

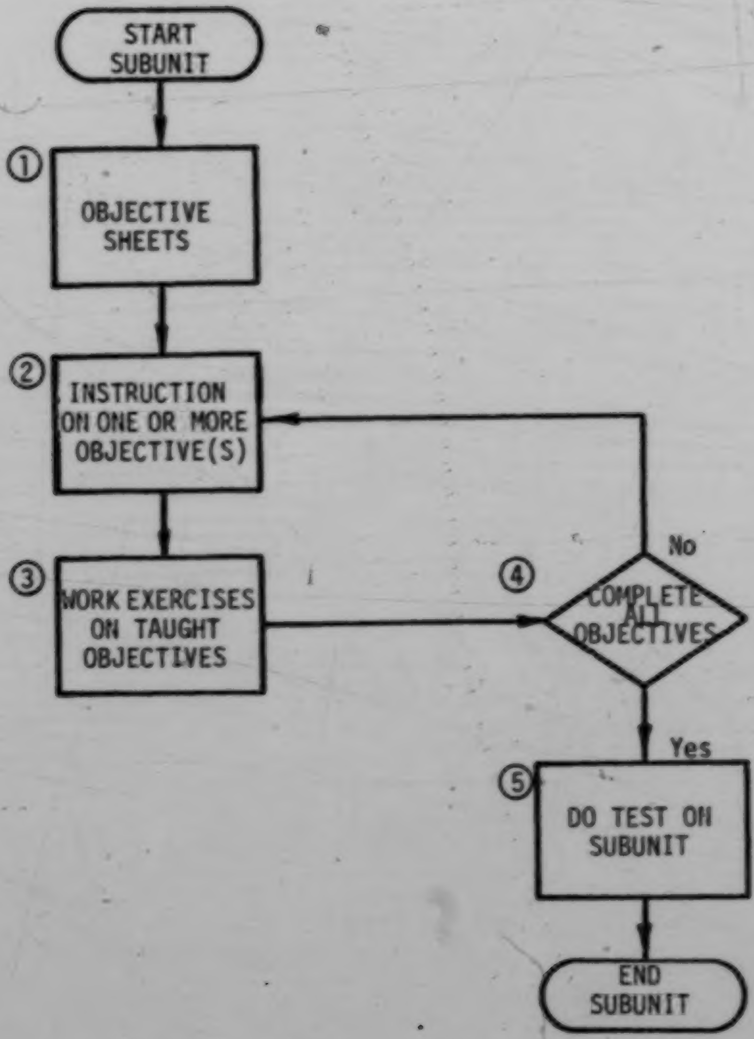

START SUBUNIT
①
OBJECTIVE SHEETS
②
INSTRUCTION ON ONE OR MORE OBJECTIVE(S)
③
WORK EXERCISES ON TAUGHT OBJECTIVES
④
COMPLETE ALL OBJECTIVES
No
Yes
⑤
DO TEST ON SUBUNIT
END SUBUNIT

FIGURE 3

FLOWCHART FOR ONE SUBUNIT OF

INSTRUCTION USING THE NON-MASTERY TREATMENT

TESTS AND OTHER INSTRUMENTS USED

Summative and Postsummative Tests

During the first class period immediately following the completion of subunit III all students wrote the summative test. Students who were unable to finish the test during the class period were given whatever time they required to complete it. The tests were graded and returned to the students.

During the next two weeks the students studied new material which made extensive use of the skills which they had acquired during their work on the unit. No new instruction was given on the basic learning tasks of the unit. Both teachers followed their usual instructional procedures. Neither researcher was present during these two weeks.

The postsummative test was written by all the students at the end of this two week period.

Attitude Toward Mathematics Test

Identical forms of this test were completed by all students prior to the experiment and following the writing of the summative test.

Opinionnaire Regarding Mastery Learning Approach

Following the writing of the Attitude Toward Mathematics and Self-Concept of Ability posttests, the MMM students were asked to express their opinions regarding the mastery learning approach. They were also asked to suggest ways in which the approach could be improved.

COOPERATION VERSUS COMPETITION

Since this aspect of the study was of a lesser importance, classroom observations were made during a 20-minute segment of one class period for each of the six classes and during each of the three subunits.

During each 20-minute period, records were made of the form of instruction carried on by the teacher and the extent of the help given to the students by their peers, the teachers or the researchers.

DATA COLLECTED

Although 153 students participated in the experiment, it was possible to collect complete data on only 141 students. The following data relating to these 141 students were obtained either from the school records or by the researchers:

1. Cooperative School and College Ability Test scores (SCAT).
2. Sequential Tests of Educational Progress scores (STEP).
3. Mathematics grades awarded by the teacher on the most recent report.
4. Attitude Toward Mathematics scores on the pretest and posttest.
5. General Self-Concept of Ability scores on the pretest and posttest.
6. General Importance of Grades scores on the pretest and posttest.
7. Self-Concept of Mathematics Ability scores on the pretest and posttest.

8. Importance of Mathematics Grades Scores on the pretest and posttest.
9. Summative test scores.
10. Postsummative test scores.

The following data were also gathered:

11. The reactions of the MMM students toward the mastery learning method of teaching mathematics.,
12. Observations of the instructional methods and the help given to students by their peers, by the teacher or by the researcher(s) during sample classroom observations.

Appendix F contains the scores of the 141 students on the first ten of the above.

RESEARCH QUESTIONS AND ANALYSES USED

The data gathered from the testing program described in the preceding pages were used in statistical analyses to test the following hypotheses. The level of significance applied throughout the study was $\alpha = 0.05$.

QUESTION I

What differences exist in student characteristics between students who attain the achievement criterion and those who do not attain the achievement criterion

(a) in a mastery learning situation (MMM)?

(b) in a non-mastery learning situation (NNN)?

It was desired to ascertain if there were differences in student characteristics between the students who attained the score of 80 per cent or higher on the summative test and the students whose score was less than 80 per cent in each of the mastery and non-mastery learning situations.

The null hypotheses for the MMM group of students are as follows:

Null Hypothesis 1(a) (1):

There is no significant difference in the distributions of SCAT (Verbal) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (2):

There is no significant difference in the distributions of SCAT (Non-Verbal) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (3):

There is no significant difference in the distributions of SCAT (Total) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (4):

There is no significant difference in the distributions of STEP scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (5):

There is no significant difference in the distributions of Teacher Evaluation of Achievement for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (6):

There is no significant difference in the distributions of Attitude Toward Mathematics pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (7):

There is no significant difference in the distributions of General Self-Concept of Ability pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (8):

There is no significant difference in the distributions of General Importance of Grades pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (9):

There is no significant difference in the distributions of Self-Concept of Mathematics Ability pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a) (10):

There is no significant difference in the distributions of Importance of Mathematics Grades pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

The null hypotheses for the NNN group of students are as follows:

Null Hypothesis 1(b) (1):

There is no significant difference in the distributions of SCAT (Verbal) scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (2):

There is no significant difference in the distributions of SCAT (Non-Verbal) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(b) (3):

There is no significant difference in the distributions of SCAT (Total) scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (4):

There is no significant difference in the distribution of STEP scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (5):

There is no significant difference in the distribution of Teacher Evaluation of Achievement for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (6):

There is no significant difference in the distributions of Attitude Toward Mathematics pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (7):

There is no significant difference in the distributions of General Self-Concept of Ability pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (8):

There is no significant difference in the distributions of

General Importance of Grades pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (9):

There is no significant difference in the distributions of Self-Concept of Mathematics ability pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b) (10):

There is no significant difference in the distributions of Importance of Mathematics grades pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Each of the above hypotheses related to Question I was tested by means of a standard analysis of variance applying the fixed effect model for equal or unequal observations in each group.

In addition, in order to protect against the effects of positive correlations among the variables (some of which are subtests) a multivariate analysis was conducted. The program used (a two-sample Hotelling T^2 test) compares sample mean vectors. Morrison (1967) provides a good description of this form of multivariate analysis.

If two independent samples of size N_1 and N_2 are selected from two p-variate normal populations $N(\mu_1, \Sigma)$ and $N(\mu_2, \Sigma)$, respectively, where μ_1 and μ_2 are mean vectors and Σ is common but unknown dispersion or covariance matrix assumed to be positive definite, then the program tests the null hypothesis that,

(1) $H_0: \underline{\mu}_1 - \underline{\mu}_2 = \underline{\delta}$, where $\underline{\delta}$ is a constant vector, usually assumed to be a null vector but not necessarily so.

In the case of sample mean vectors $\underline{\bar{y}}_1$ and $\underline{\bar{y}}_2$, sample within-group dispersions $\underline{S}_1$ and $\underline{S}_2$, the statistic

(2) $$T^2 = \frac{N_1 N_2}{N_1 + N_2} (\underline{\bar{y}}_1 - \underline{\bar{y}}_2 - \underline{\delta}) S^{-1} (\underline{\bar{y}}_1 - \underline{\bar{y}}_2 - \underline{\delta})$$

is distributed as Hotelling T^2 with parameter p and $N_1 + N_2 - p - 1$ where $\underline{S}$ is pooled sample dispersion matrix obtained by,

(3) $\underline{S} = (\underline{S}_1 + \underline{S}_2)/(N_1 + N_2 - 2)$.

If the hypothesis (1) is rejected, a check may be done to determine which variate or linear combination of the variates leads to the rejection of the hypothesis by multiple comparisons or simultaneous confidence intervals of the linear compound of the type $\underline{a}'\underline{\delta}$. This check requires that the weight vector $\underline{a}$ to indicate interested linear compound of variates, and desired alpha level for the confidence intervals be specified.

QUESTION II

What student characteristics can be used to predict mathematics achievement scores in a mastery learning setting?

It was desired to ascertain what student characteristics, if any, were useful in predicting mathematics achievement scores in a mastery learning setting (i.e., MMM setting).

The null hypotheses relative to Question II are as follows:

Null Hypothesis 2(a):

The correlation coefficient between a MMM student characteristic and achievement score is not significantly different from zero.

Null Hypothesis 2(b):

The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion achievement variable for a MMM student is not significantly different from zero.

In analysing the above hypotheses the following procedures were used.

For Null Hypothesis 2(a), tables of Pearson product - moment correlations between student characteristic scores and the criterion achievement scores were prepared for each of the MMM and NNN groups. The tables permitted an inspection of the "closeness" of linear relations between the predictor variables and the criterion achievement scores.

The correlation coefficients and the corresponding probabilities that the correlations in the population from which the sample was drawn are equal to zero, were used to test Null Hypothesis 2(a).

For Null Hypothesis 2(b), multiple regression analysis was used to construct a linear combination of independent variables (student characteristics) which "best" predicts the dependent or criterion variable (achievement scores).

A stepwise regression procedure was used to arrive at a linear regression equation of the form:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 \ldots + b_nx_n + e$$

in which

y is the dependent variable;

$x_1, x_2, x_3, \ldots, x_n$ are the independent variables;

$b_0, b_1, b_2, \ldots, b_n$ are the coefficients that produce the "best" equation; and e is the error term (the difference between the predicted and actual values of the dependent variable). The "best" equation is defined by the coefficients, b_0, b_1, . . ., b_n, that make the sum of e^2 a minimum for a particular series of criterion values and predictor values obtained from a given sample.

The computer program used for this part of the study employed the methods of determinants as outlined by Draper and Smith (1966). The advantage that this program has over more usual methods of regression analysis is that one may specify the level of significance for adding and deleting predictor variables.

The procedure involves the re-examination at every stage of the regression of the variables incorporated into the model in previous stages. A variable which may have been the best single variable to enter at an early stage, may, at a later stage, be superfluous because of the relationship between it and other variables now in the regression. As a check on this, the partial F criterion for each variable in the regression at any stage of calculation is evaluated and compared with a preselected percentage point of the appropriate F distribution. This provides a judgment on the contribution made by each variable as though it had been the most recent variable entered irrespective of its actual point of entry into the model. Any variable which provides a nonsignificant contribution is removed from the model. This process is continued until no more variables will be admitted to the equation and no more are rejected.

In this study, a level of significance of $\alpha = 0.05$ was specified for adding and deleting predictor variables.

QUESTION III

What student characteristics can be used to predict mathematics achievement scores in a non-mastery learning setting?

It was desired to ascertain what characteristics, if any, were useful in predicting mathematics achievement scores in a non-mastery learning setting (i.e., NNN setting).

The null hypotheses relative to question III are as follows:

Null Hypothesis 3(a):

The correlation coefficient between a NNN student characteristic and achievement score is not significantly different from zero.

Null Hypothesis 3(b):

The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion achievement variable for a NNN student is not significantly different from zero.

The analytical procedures used to test the above hypotheses were the same as described for Null Hypotheses 2(a) and 2(b).

QUESTION IV

To what extent does the mastery learning method of teaching mathematics affect the following student characteristics:

(a) student attitude toward mathematics?

(b) student general self-concept of ability?

(c) importance that the student attaches to grades in general?

(d) student self-concept of mathematics ability?

(e) importance that the student attaches to mathematics grades?

The null hypotheses relative to Question IV are as follows. The four groups referred to are MMM group, NMM group, NNM group and the NNN group of students.

Null Hypothesis 4(a):

There is no significant difference in the gains of the Attitude Toward Mathematics scores within each of the four groups of students.

Null Hypothesis 4(b):

There is no significant difference in the gains of the General Self-Concept of Ability scores within each of the four groups of students.

Null Hypothesis 4(c):

There is no significant difference in the gains of General Importance of Grades scores within each of the four groups of students.

Null Hypothesis 4(d):

There is no significant difference in the gains of the Self-Concept of Mathematics Ability scores within each of the four groups of students.

Null Hypothesis 4(e):

There is no significant difference in the gains of Importance of Mathematics Grades scores within each of the four groups of students.

The statistical program used to test Null Hypotheses 4(a), 4(b), 4(c), 4(d) and 4(c) was the One-Way Analysis of Variance With Repeated Measures. Total variation is broken into between people variation and within people variation. The latter is broken into between treatment variation and residual variation.

QUESTION V

Under mastery learning conditions, to what extent do students develop cooperation as opposed to competition in their learning?

This aspect of the experiment was undertaken as a case study in view of the numerous problems in establishing controls. For example;

1. It was not possible to observe each of the six classes in operation for more than one 20-minute period during each of the three subunits. For one researcher to spend more time in observing may have endangered other more crucial aspects of the experiment.
2. Since the researcher concerned could not observe more than one class at one time, random selection of the periods of observation was impracticable.
3. Since the classes observed were conducted in their usual settings, some classes were observed in science laboratories and some in regular classrooms. The science laboratory setting may have encouraged students to work together, since students were seated in groups around tables. The regular classroom setting may have inhibited students' working together, since students were usually seated at individual desks arranged in columns.
4. Student attendance varied from period to period and also within periods as students were called away or were involved in other activities.

5. Occasionally, the teacher or the observer's colleague was unable to be present for a portion of, or for all of the 20-minute period of observation.

Procedures Adopted

The observer was situated towards the rear of the room or laboratory. From this vantage point he could obtain a clear view of class activities. Thus, it was possible for him to discriminate between students' cooperating on their mathematics work and their engaging in gossip.

Once housekeeping and routine matters (e.g., recording attendance) had been taken care of and instruction had started, the 20-minute period of observation commenced. Observation and recording ceased at the end of each 20-minute period.

Appendix E contains a copy of the Classroom Observation Record. The 20-minute period was divided into modules of one minute and a record kept of activity during each of the one-minute modules. The numbers and types of tutoring units (peer tutoring, teacher tutoring and researcher tutoring) were recorded for each one-minute module. Other activity, for example, lectures or demonstrations by the teacher, was recorded in terms of one-minute modules.

STUDENT OPINION REGARDING THE MASTERY LEARNING APPROACH

Following the writing of the Attitude Toward Mathematics and the Self-Concept of Ability and associated posttests, the MMM students were asked to express their opinions regarding the mastery learning approach. They were also requested to suggest ways in which the method could be improved. The questionnaire may be found in Appendix E.

SUMMARY

This chapter has consisted of a description of the experimental design, a statement of hypotheses, and an outline of the statistical procedures followed in analysing the data collected in the study. The results of the analyses are presented in the chapter which follows.

CHAPTER V

RESULTS OF THE STUDY

INTRODUCTION

It is the purpose of this chapter to present detailed results of the study described in Chapter IV. To facilitate the reporting of results, each null hypothesis will be stated, an analysis of the data will follow and a decision will be made regarding the acceptance or rejection of the null hypothesis. The level of significance applied throughout the study is $\alpha = 0.05$.

The scores on the tests described in Chapter IV, which form the basis for the results of this chapter, are contained in Appendix F. Although 153 students participated in the experiment, incomplete data were obtained for 12 students. Data for these 12 students have been omitted in the various analyses of the study. Appendix F contains complete data for the other 141 students.

Before considering the results of the analyses relating to the numerous hypotheses stated in this chapter, it is desirable to examine the student characteristics of the two randomly assigned groups. Some of the characteristic scores were obtained from the school records; for the other characteristics the pretest scores were used.

Table III displays the means and standard deviations of student characteristic scores obtained for the 48 students who received mastery learning treatment (MMM) and the 45 students who experienced the non-mastery learning treatment (NNN). The t-ratios and associated probab-

TABLE III

MEANS, STANDARD DEVIATIONS (S.D.) AND T-RATIOS FOR TEST SCORES OF STUDENTS IN MMM[1] AND NNN[2] GROUPS

Variable	Treatment Group	Mean	S.D.	T-Ratio	P	Adjusted T-Ratio	Adjusted P
X 1 : SCAT (Verbal)	MMM NNN	54.50 52.31	24.97 24.36	0.427	0.670	0.428	0.670
X 2 : SCAT (Non-Verbal)	MMM NNN	63.65 56.00	25.86 24.20	1.470	0.145	1.473	0.144
X 3 : SCAT (Total)	MMM NNN	59.71 54.20	25.40 22.86	1.097	0.276	1.100	0.274
X 4 : STEP	MMM NNN	52.94 52.38	26.39 28.06	0.099	0.921	0.099	0.921
X 5 : Teacher Evaluation	MMM NNN	3.19 3.22	1.14 0.93	0.160	0.873	0.160	0.872
X 6 : Attitude Toward Mathematics Pretest	MMM NNN	87.13 88.13	15.21 16.95	0.302	0.763	0.301	0.764
X 7 : Self-Concept of Ability Pretest	MMM NNN	29.06 28.73	4.81 5.19	0.317	0.752	0.317	0.752
X 8 : General Importance of Grades Pretest	MMM NNN	24.90 24.67	3.40 4.01	0.255	0.766	0.296	0.768
X 9 : Self-Concept of Mathematics Ability Pretest	MMM NNN	27.27 26.64	6.28 5.99	0.492	0.624	0.493	0.624
X10 : Importance of Mathematics Grades Pretest	MMM NNN	21.10 20.76	3.52 3.65	0.469	0.640	0.468	0.641

[1] MMM = mastery learning treatment: number of MMM students = 48

[2] NNN = non-mastery learning treatment: number of NNN students = 45

ilities which were calculated to test for significance of a difference between the group means are included. To allow for variables with unequal variances, Welch's approximation to "t" and the corresponding probabilities were calculated and are also included in the table.

Since the calculated probabilities range from 0.144 to 0.921 (adjusted) it may be assumed that there are no significant differences between the two randomly assigned groups as far as student characteristics are concerned.

NULL HYPOTHESES

Following are the analyses of the five questions and the respective hypotheses posed in earlier chapters. The level of significance is $\alpha = 0.05$ in each case.

QUESTION I

What differences exist in student characteristics between students who attain the achievement criterion and those who do not attain the achievement criterion:

(a) in a mastery learning situation (MMM)?

(b) in a non-mastery learning situation (NNN)?

The hypotheses relative to Question I were tested only for the students who experienced either the mastery learning conditions throughout the four weeks of the experiment (i.e., the 48 MMM students) or for the students who experienced the non-mastery learning conditions during the four weeks of the experiment (i.e., the 45 NNN students). Of the 48 MMM students, 20 students attained the mastery criterion of 80 per cent or higher on the summative test. Of the 45 NNN students, 10 students attained the mastery criterion of 80 per cent or higher on the summative test.

The following Null Hypotheses, 1(a)(1), 1(a)(2), 1(a)(3), 1(a)(4), 1(a)(5), 1(a)(6), 1(a)(7), 1(a)(8), 1(a)(9) and 1(a)(10), relate to the mastery learning situation (MMM):

Null Hypothesis 1(a)(1):

There is no significant difference in the distributions of SCAT (Verbal) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(2):

There is no significant difference in the distributions of SCAT (Non-Verbal) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(3):

There is no significant difference in the distributions of SCAT (Total) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(4):

There is no significant difference in the distributions of STEP scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(5):

There is no significant difference in the distributions of Teacher Evaluation of Achievement for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(6):

There is no significant difference in the distributions of

Attitude Toward Mathematics pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(7):

There is no significant difference in the distributions of General Self-Concept of Ability pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(8):

There is no significant difference in the distributions of General Importance of Grades pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(9):

There is no significant difference in the distributions of Self-Concept of Mathematics Ability pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(a)(10):

There is no significant difference in the distributions of Importance of Mathematics Grades pretest scores for MMM students who attained criterion and MMM students who did not attain criterion.

Tables IV, V, VI, VII, VIII, IX, X, XI, XII and XIII contain details of the analyses of Null Hypotheses 1(a)(1), 1(a)(2), 1(a)(3), 1(a)(4), 1(a)(5), 1(a)(6), 1(a)(7), 1(a)(8), 1(a)(9) and 1(a)(10),

TABLE IV

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SCAT (VERBAL) TEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	5529.88	10.70	.002
Within Groups	46	517.00		

TABLE V

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SCAT (NON-VERBAL) TEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	6345.38	11.63	.001
Within Groups	46	545.47		

TABLE VI

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SCAT (TOTAL) TEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	7757.19	15.81	.0002
Within Groups	46	490.76		

TABLE VII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF STEP TEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	10216.88	20.87	.00004
Within Groups	46	489.52		

TABLE VIII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF TEACHER EVALUATION OF ACHIEVEMENT FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	17.41	18.23	.0001
Within Groups	46	.95		

TABLE IX

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF ATTITUDE TOWARD MATHEMATICS PRETEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	140.69	.60	.441
Within Groups	46	233.19		

TABLE X

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF GENERAL SELF-CONCEPT OF ABILITY PRETEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	103.50	4.83	.033
Within Groups	46	21.42		

TABLE XI

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF GENERAL IMPORTANCE OF GRADES PRETEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	.10	.01	.928
Within Groups	46	11.79		

TABLE XII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SELF-CONCEPT OF MATHEMATICS ABILITY PRETEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	155.43	4.22	.046
Within Groups	46	36.87		

TABLE XIII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF IMPORTANCE OF MATHEMATICS GRADES PRETEST SCORES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	.37	.03	.865
Within Groups	46	12.65		

An examination of Tables IV, V, VI, VII, VIII, IX, X, XI, XII and XIII will indicate that differences (at the $\alpha = 0.05$ level) exist between the distributions of certain scores for the MMM students who attained criterion and those MMM students who did not attain criterion on the summative test. The significant differences are as follows:

Variate	Probability (p)
SCAT (Verbal)	0.002
SCAT (Non-Verbal)	0.001
SCAT (Total)	0.0002
STEP	0.00003
TEACHER EVALUATION OF ACHIEVEMENT	0.0001
GENERAL SELF-CONCEPT OF ABILITY	0.033
SELF-CONCEPT OF MATHEMATICS ABILITY	0.046

In order to guard against the effects of positive correlations among the student characteristics (some of which are subtests) the two-sample Hotelling T^2 test described in Chapter IV was performed. The level of significance was $\alpha = 0.05$.

Table XIV shows the means, and differences in means of the student characteristic scores of the two samples. Sample 1 consists of the 20 MMM students who attained criterion; Sample 2 consists of the 28 MMM students who did not attain criterion.

TABLE XIV

MEANS AND DIFFERENCES OF MEANS OF VARIATES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

VARIATE	SAMPLE 1 MEAN $\overline{X}_1$	SAMPLE 2 MEAN $\overline{X}_2$	DIFFERENCES IN MEANS $\overline{X}_1 - \overline{X}_2$
SCAT (Verbal)	67.20	45.43	21.77
SCAT (Non-Verbal)	77.25	53.93	23.32
SCAT (Total)	74.75	48.96	25.79
STEP	70.20	40.61	29.59
TEACHER EVALUATION	3.90	2.68	1.22
ATTITUDE TOWARD MATH	89.15	85.68	3.47
GEN. SELF-CONCEPT OF ABILITY	24.95	24.86	0.09
SELF-CONCEPT OF MATH ABILITY	29.40	25.75	3.65
IMPORTANCE OF MATH GRADES	21.00	21.18	-0.18

The null hypothesis,

$$H_0 : \mu_1 - \mu_2 = 0$$

that the samples arose from populations with a common mean vector, was tested at the $\alpha = 0.05$ level.

The two-sample Hotelling T^2 statistic had the value 33.112; the associated F-ratio was 2.663 with degrees of freedom 10 and 37, and the probability was 0.015.

Thus, the null hypothesis may be rejected as far as the "global" situation is concerned.

The analysis was performed once more to ascertain which single variates may have led to the rejection of the null hypothesis, i.e., to ascertain which student characteristic means were significantly different.

Table XV shows the results of the Hotelling T^2 analysis for each of the student characteristic variates. The degrees of freedom are 10 and 37, and the level of significance is $\alpha = 0.05$ in each case.

TABLE XV

HOTELLING T^2 ANALYSIS OF DIFFERENCES OF MEANS OF VARIATES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

VARIATE	T^2	F-RATIO	PROBABILITY (p)	95% CONF INTERVAL LOWER BOUND	95% CONF INTERVAL UPPER BOUND
SCAT (Verbal)	10.696	0.860	0.577	-12.22	55.77
SCAT (Non-Verbal)	11.633	0.936	0.513	-11.60	58.24
SCAT (Total)	15.807	1.271	0.282	- 7.34	58.91
STEP	20.871	1.679	0.123	- 3.49	62.67
TEACHER EVALUATION	18.235	1.467	0.191	- 0.24	2.68
ATTITUDE TOWARD MATH	0.603	0.048	1.000	-19.36	26.30
GEN. SELF-CONCEPT OF ABILITY	4.832	0.389	0.944	- 3.94	9.90
GEN. IMPORTANCE OF GRADES	0.009	0.001	1.000	- 5.04	5.23
SELF-CONCEPT OF MATH ABILITY	4.216	0.339	0.964	- 5.43	12.73
IMPORTANCE OF MATH GRADES	0.029	0.002	1.000	- 5.50	5.14

From Table XV it is evident that there are no single variates which have led to the rejection of the null hypothesis.

The Hotelling T^2 analysis was performed again to ascertain if any conceptually feasible linear combinations of variates may have led to the rejection of the null hypothesis.

It seemed feasible from a conceptual standpoint to conduct an Hotelling T^2 analysis of the following linear combinations of variates:

1. SCAT (Total).
2. STEP; Teacher Evaluation of Achievement
3. Attitude Toward Mathematics; General Importance of Grades; Importance of Mathematics Grades.
4. General Self-Concept of Ability; General Importance of Grades.

Table XVI shows the results of the Hotelling T^2 analysis for the variates combined as shown above.

TABLE XVI

HOTELLING T^2 ANALYSIS OF LINEAR COMBINATIONS OF VARIATES FOR MMM STUDENTS WHO ATTAINED CRITERION AND MMM STUDENTS WHO DID NOT ATTAIN CRITERION

COMBINATION OF VARIATES	T^2	F-RATIO	PROBABILITY (p)	95% CONF INTERVAL	
				LOWER BOUND	UPPER BOUND
SCAT (Total)	15.807	1.271	0.282	- 7.34	58.91
STEP; TEACHER EVALUATION	21.677	1.744	0.107	- 1.49	32.31
ATTITUDE TOWARD MATH GEN. IMPORTANCE OF GRADES IMPORTANCE OF MATH GRADES	0.339	0.027	1.000	-86.78	109.13
GEN. SELF-CONCEPT OF ABILITY GEN. IMPORTANCE OF GRADES	5.030	0.405	0.936	- 4.23	10.86

From Table XVI it is evident that none of the linear combinations of variates that were chosen have led to the rejection of the null hypothesis.

The following Null Hypotheses 1(b)(1), 1(b)(2), 1(b)(3), 1(b)(4), 1(b)(5), 1(b)(6), 1(b)(7), 1(b)(8), 1(b)(9) and 1(b)(10) relate to the non-mastery learning situation (NNN):

Null Hypothesis 1(b)(1):

There is no significant difference in the distributions of SCAT (Verbal) scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(2):

There is no significant difference in the distributions of SCAT (Non-Verbal) scores for MMM students who attained criterion and MMM students who did not attain criterion.

Null Hypothesis 1(b)(3):

There is no significant difference in the distributions of SCAT (Total) scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(4):

There is no significant difference in the distributions of STEP scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(5):

There is no significant difference in the distributions of Teacher Evaluation of Achievement for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(6):

There is no significant difference in the distributions of Attitude Toward Mathematics pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(7):

There is no significant difference in the distributions of General Self-Concept of Ability pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(8):

There is no significant difference in the distributions of General Importance of Grades pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(9):

There is no significant difference in the distributions of Self-Concept of Mathematics Ability pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Null Hypothesis 1(b)(10):

There is no significant difference in the distributions of Importance of Mathematics Grades pretest scores for NNN students who attained criterion and NNN students who did not attain criterion.

Tables XVII, XVIII, XIX, XX, XXI, XXII, XXIII, XXIV, XXV and XXVI contain details of the analyses of Null Hypotheses 1(b)(1), 1(b)(2), 1(b)(3), 1(b)(4), 1(b)(5), 1(b)(6), 1(b)(7), 1(b)(8), 1(b)(9) and 1(b)(10), respectively.

TABLE XVII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SCAT (VERBAL) TEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	1667.63	2.93	0.094
Within Groups	43	568.65		

TABLE XVIII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SCAT (NON-VERBAL) TEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	5724.06	12.28	0.001
Within Groups	43	466.18		

TABLE XIX

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SCAT (TOTAL) TEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	4028.00	9.13	0.004
Within Groups	43	441.19		

TABLE XX

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF STEP TEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	4900.06	7.08	0.011
Within Groups	43	691.64		

TABLE XXI

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF TEACHER EVALUATION OF ACHIEVEMENT FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	7.78	11.15	0.002
Within Groups	43	0.70		

TABLE XXII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF ATTITUDE TOWARD MATHEMATICS PRETEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	2228.94	9.20	0.004
Within Groups	43	242.29		

TABLE XXIII

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF GENERAL SELF-CONCEPT OF ABILITY PRETEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	202.30	8.84	0.005
Within Groups	43	22.90		

TABLE XXIV

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF GENERAL IMPORTANCE OF GRADES PRETEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	151.55	11.71	0.001
Within Groups	43	12.94		

TABLE XXV

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF SELF-CONCEPT OF MATHEMATICS ABILITY PRETEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	411.24	15.15	0.0003
Within Groups	43	27.14		

TABLE XXVI

SUMMARY TABLE FOR ANALYSIS OF VARIANCE OF IMPORTANCE OF MATHEMATICS GRADES PRETEST SCORES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

Source	d.f.	MS	F	p
Between Groups	1	70.66	5.89	0.019
Within Groups	43	11.99		

An examination of Tables XVII, XVIII,XIX, XX, XXI, XXII, XXIII, XXIV, XXV and XXVI will indicate that differences (at the $\alpha = 0.05$ level) exist between the distributions of certain scores for the NNN students who attained criterion and those NNN students who did not attain criterion on the summative test. The significant differences are as follows:

Variate	Probability (p)
SCAT (Non-Verbal)	0.001
SCAT (Total)	0.004
STEP	0.011
TEACHER EVALUATION OF ACHIEVEMENT	0.002
ATTITUDE TOWARD MATHEMATICS	0.004
GENERAL SELF-CONCEPT OF ABILITY	0.005
GENERAL IMPORTANCE OF GRADES	0.001
SELF-CONCEPT OF MATHEMATICS ABILITY	0.0003
IMPORTANCE OF MATHEMATICS GRADES	0.019

In order to guard against the effects of positive correlations among the student characteristics, the two-sample Hotelling T^2 test was performed. The level of significance was $\alpha = 0.05$.

Table XXVII shows the means, and differences in means of the student characteristic scores of the two samples. Sample 1 consists of the 10 NNN students who attained criterion; Sample 2 consists of the 35 NNN students who did not attain criterion.

TABLE XXVII

MEANS AND DIFFERENCES OF MEANS OF VARIATES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

VARIATE	SAMPLE 1 MEAN $\overline{X}_1$	SAMPLE 2 MEAN $\overline{X}_2$	DIFFERENCES IN MEANS $\overline{X}_1 - \overline{X}_2$
SCAT (Verbal)	63.70	49.06	14.64
SCAT (Non-Verbal)	77.10	49.97	27.13
SCAT (Total)	71.90	49.14	22.76
STEP	71.90	46.80	25.10
TEACHER EVALUATION	4.00	3.00	1.00
ATTITUDE TOWARD MATH	101.30	84.37	16.93
GEN. SELF-CONCEPT OF ABILITY	32.70	27.60	5.10
GEN. IMPORTANCE OF GRADES	28.10	23.69	4.41
SELF-CONCEPT OF MATH ABILITY	32.30	25.03	7.27
IMPORTANCE OF MATH GRADES	23.10	20.09	3.01

The null hypothesis,

$$H_o : \mu_1 - \mu_2 = 0$$

that the samples arose from populations with a common mean vector, was tested at the $\alpha = 0.05$ level.

The two-sample Hotelling T^2 statistic had the value 46.067; the associated F-ratio was 3.643 with degrees of freedom 10 and 34, and the probability was 0.002.

Thus, the null hypothesis may be rejected as far as the "global" situation is concerned.

The analysis was performed once more to ascertain which variate led to the rejection of the null hypothesis, i.e., to ascertain which student characteristic means were significantly different.

Table XXVIII shows the results of the Hotelling T^2 analysis for each of the student characteristic variates. The degrees of freedom are 10 and 34, and the level of significance is $\alpha = 0.05$ in each case.

TABLE XXVIII

HOTELLING T^2 ANALYSIS OF DIFFERENCES OF MEANS OF VARIATES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

VARIATE	T^2	F-RATIO	PROBABILITY (p)	95% CONF INTERVAL	
				LOWER BOUND	UPPER BOUND
SCAT (Verbal)	2.933	0.232	0.991	-29.66	58.95
SCAT (Non-Verbal)	12.279	0.971	0.486	-12.99	67.25
SCAT (Total)	9.130	0.722	0.698	-16.27	61.78
STEP	7.085	0.560	0.834	-23.76	73.96
TEACHER EVALUATION	11.148	0.881	0.559	- 0.55	2.55
ATTITUDE TOWARD MATH	9.200	0.727	0.694	-11.99	45.85
GEN. SELF-CONCEPT OF ABILITY	8.836	0.699	0.719	- 3.79	13.99
GEN. IMPORTANCE OF GRADES	11.712	0.926	0.522	- 2.27	11.10
SELF-CONCEPT OF MATH ABILITY	15.152	1.198	0.327	- 2.41	16.95
IMPORTANCE OF MATH GRADES	5.893	0.466	0.900	- 3.42	9.45

From Table XXVIII it is evident that there are no single variables which have led to the rejection of the null hypothesis.

The analysis was next conducted on the same combinations of variates as in the MMM situation.

Table XXIX shows details of the Hotelling T^2 analysis for the combinations of variates that were chosen.

TABLE XXIX

HOTELLING T^2 ANALYSIS OF LINEAR COMBINATIONS OF VARIATES FOR NNN STUDENTS WHO ATTAINED CRITERION AND NNN STUDENTS WHO DID NOT ATTAIN CRITERION

COMBINATION OF VARIATES	T^2	F-RATIO	PROBABILITY (p)	95% CONF INTERVAL	
				LOWER BOUND	UPPER BOUND
SCAT (Total)	9.130	0.722	0.698	-16.27	61.78
STEP; TEACHER EVALUATION	7.554	0.597	0.805	-11.55	37.65
ATTITUDE TOWARD MATH GEN. IMPORTANCE OF GRADES IMPORTANCE OF MATH GRADES	11.177	0.884	0.557	-44.20	204.96
GEN. SELF-CONCEPT OF ABILITY GENERAL IMPORTANCE OF GRADES	13.701	1.083	0.401	- 2.47	14.85

From Table XXIX, it is evident that none of the linear combinations of variates that were chosen have led to the rejection of the null hypothesis.

Summary of Results of Analyses for Question I

When the hypotheses were tested by means of an analysis of variance applying the fixed effect model for equal or unequal observations in each group, the following significant differences (at the $\alpha = 0.05$ level) were found.

For the MMM situation the following significant differences were found:

Variate	Probability (p)
SCAT (Verbal)	0.002
SCAT (Non-Verbal)	0.001
SCAT (Total)	0.0002
STEP	0.0003
TEACHER EVALUATION	0.0001
GENERAL SELF-CONCEPT OF ABILITY	0.033
SELF-CONCEPT OF MATHEMATICS ABILITY	0.046

For the NNN situation the significant differences were as follows:

Variate	Probability (p)
SCAT (Non-Verbal)	0.001
SCAT (Total)	0.004
STEP	0.011
TEACHER EVALUATION	0.002
ATTITUDE TOWARD MATHEMATICS	0.004
GENERAL SELF-CONCEPT OF ABILITY	0.005
GENERAL IMPORTANCE OF GRADES	0.001
SELF-CONCEPT OF MATHEMATICS ABILITY	0.0003
IMPORTANCE OF MATHEMATICS GRADES	0.019

The differences that were significant in both the MMM and the NNN situations are as follows:

SCAT (Non-Verbal)
SCAT (Total)
STEP
TEACHER EVALUATION
GENERAL SELF-CONCEPT OF ABILITY
SELF-CONCEPT OF MATHEMATICS ABILITY

The differences that were significant in only one situation are as follows:

SCAT (Verbal) which was significantly different only in the MMM situation, and

IMPORTANCE OF MATHEMATICS GRADES which was significantly different only in the NNN situation.

When the two-sample Hotelling T^2 analysis was performed the results were as follows:

In the mastery learning situation (MMM), the analysis showed that "globally" the null hypothesis should be rejected at the $\alpha = 0.05$ level of significance. However, no single variates or conceptually feasible linear combinations of variates were established as having led to the rejection of the null hypothesis.

In the non-mastery situation (NNN) the analysis showed that "globally" the null hypothesis should be rejected at the $\alpha = 0.05$ level. However, as in the MMM situation, no single variates or conceptually feasible linear combinations of variates were established as having led to the rejection of the null hypothesis. All of the probabilities, either for single variates or for combinations of variates, were greater than 0.300.

QUESTION II

What student characteristics can be used to predict mathematics achievement scores in a mastery learning setting?

Null Hypotheses 2(a) and 2(b) relate to Question II.

Null Hypothesis 2(a):

The correlation coefficient between a MMM student characteristic and achievement score is not significantly different from zero.

This null hypothesis was repeated in the MMM setting for each of the 10 student characteristics and stated separately for two sets of achievement scores (summative and postsummative tests).

Table XXX gives the Pearson correlation coefficients showing the "closeness" of a linear relationship between the criterion achievement test results and the 10 student characteristics for the MMM group. The symbols X_1, X_2, etc., represent the variables as indicated in Table III. The factors which have significant correlations (at the $\alpha = 0.05$ level) with the achievement tests are indicated by an asterisk (*).

TABLE XXX

CORRELATIONS OF STUDENT CHARACTERISTIC SCORES WITH SUMMATIVE AND POSTSUMMATIVE TEST SCORES FOR MMM STUDENTS

Test	X_1	X_2	X_3	X_4	X_5
Summative	.565*	.560*	.653*	.712*	.739*
Postsummative	.545*	.482*	.588*	.664*	.721*
Test	X_6	X_7	X_8	X_9	X_{10}
Summative	.291*	.563*	.155	.481*	.083
Postsummative	.287*	.561*	.061	.447*	-.040

*$p < .05$

From Table XXX it is evident that the factor most highly correlated with achievement is X_5 (Teacher Evaluation). This implies that students who received high scores in the last home report tended to have a high score on the summative and postsummative tests in the mastery learning situation.

Factor X_4 (STEP) bore the next higher correlation with achievement. The factors X_3 [SCAT (Total)], X_2 [SCAT (Non-Verbal)], X_7 (General Self-Concept of Ability), X_1 [SCAT (Verbal)], X_9 (Self-Concept of Mathematics Ability) and X_6 (Attitude Toward Mathematics) bore a significant relationship with both criterion achievement tests.

Zero, or almost zero, correlation existed between achievement and factors X_8(General Importance of Grades) and X_{10} (Importance of Mathematics Grades).

Null Hypothesis 2(b):

The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion achievement variable for a MMM student is not significantly different from zero.

The stepwise regression analysis outlined in Chapter IV resulted in regression equations for each of the summative and postsummative achievement tests. Information pertaining to the regression equations for the MMM students is summarized in Tables XXXI and XXXII

TABLE XXXI

MULTIPLE REGRESSION EQUATION SIGNIFICANT PREDICTORS (0.05 LEVEL) OF SUMMATIVE TEST SCORES FOR MMM STUDENTS

Variable	Standard Weight	Weight	Standard Error
X_4	0.41	0.15	0.04
X_5	0.48	4.04	0.96

Constant = - 2.24

Best Predictive Equation:

Y Summative = $- 2.24 + 0.15X_4 + 4.04X_5$

Percent Variance Accounted For = 64.67

Standard Error of Predicted Y = 5.81

X_4 = STEP (Total)

X_5 = TEACHER EVALUATION

Table XXXI displays the variables that made a significant contribution in predicting the criterion variable (achievement on the summative test) for the MMM students. Thus, the best predictors of the performance of the MMM students on the summative test were:

X_4 : STEP (Total)

X_5 : TEACHER EVALUATION OF ACHIEVEMENT

The best equation for predictive purposes was:

$Y = - 2.24 + 0.15X_4 + 4.04X_5$

Almost 65 per cent of the variance of the criterion variable was accounted for by the predictors. The squared multiple correlation is significantly different from zero since the F_R value of 41.187 is greater than the critical F value of 3.20 (0.05 level) for 2 and 45 degrees of freedom. Thus, Null Hypothesis 2(b) is rejected as far as the summative test achievement is concerned.

The regression equation would predict high summative test scores for MMM students with high STEP (Total) scores and high Teacher Evaluation of Achievement scores.

TABLE XXXII

MULTIPLE REGRESSION EQUATION SIGNIFICANT PREDICTORS (0.05 LEVEL) OF POSTSUMMATIVE TEST SCORES FOR MMM STUDENTS

Variable	Standard Weight	Weight	Standard Error
X_4	0.35	0.11	0.04
X_5	0.50	3.75	0.92

Constant = 2.87

Best Predictive Equation:

Y Postsummative = $2.87 + 0.11X_4 + 3.75X_5$

Per cent Variance Accounted For = 59.23

Standard Error of Predicted Y = 5.81

X_4 = STEP (Total)

X_5 = TEACHER EVALUATION OF ACHIEVEMENT

Table XXXII displays the variables that made a significant contribution in predicting the criterion variable (achievement on the postsummative test) for the MMM students. Thus, the best predictors of the performance of the MMM students on the postsummative test were:

X_4 : STEP (Total)

X_5 : TEACHER EVALUATION OF ACHIEVEMENT

The best equation for predictive purposes was:

$Y = 2.87 + 0.11X_4 + 3.75X_5$

Almost 60 per cent of the variance of the criterion variable was accounted for by the predictors. The squared multiple correlation is significantly different from zero since the F_R value of 32.694 is greater than the critical F value of 3.20 (0.05 level) for 2 and 45 degrees of freedom. Thus, Null Hypothesis 2(b) is rejected as far as the postsummative test achievement is concerned.

The regression equation would predict high postsummative test scores for MMM students with high STEP (Total) scores and high Teacher Evaluation of Achievement scores.

Null Hypothesis 2(b) is, therefore, rejected for each of the criterion variables (summative and postsummative test scores).

The significant predictor variables in each instance were STEP (Total) and Teacher Evaluation of Achievement.

QUESTION III

What student characteristics can be used to predict mathematics achievement scores in a non-mastery learning setting?

Null Hypotheses 3(a) and 3(b) relate to Question III.

Null Hypothesis 3(a):

The correlation coefficient between a NNN student characteristic and achievement score is not significantly different from zero.

This null hypothesis was repeated in the NNN setting for each of the 10 student characteristics and stated separately for two sets of achievement scores (summative and postsummative tests).

Table XXXIII gives the Pearson correlation coefficients showing the "closeness" of a linear relationship between the criterion achievement test results and the 10 student characteristics for the NNN group. The symbols X_1, X_2, etc., represent the variables as indicated in Table III. The factors which have significant correlations (at the $\alpha = 0.05$ level) with the achievement tests are indicated by an asterisk (*).

TABLE XXXIII

CORRELATIONS OF STUDENT CHARACTERISTIC SCORES WITH SUMMATIVE AND POSTSUMMATIVE TEST SCORES FOR NNN STUDENTS

Test	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Summative	.246	.595*	.476*	.278	.652*	.622*	.497*	.431*	.716*	.456*
Postsummative	.106	.539*	.356*	.117	.579*	.383*	.238	.243	.540*	.339*

*$p < .05$

From Table XXXIII it is evident that the factors most highly correlated with both criterion achievement tests were X_9 (Self-Concept of Mathematics Ability) and X_5 (Teacher Evaluation of Achievement). This implies that students who possessed a high self-concept of their ability in mathematics tended to score highly on the summative and postsummative tests under the "regular" instructional mode. Further, students who received high scores in the last home report tended to score highly on the summative and postsummative tests under the "regular" instructional mode. Other factors which were significantly related to both criterion achievement tests were X_2 [SCAT (Non-Verbal)], X_6 (Attitude Toward Mathematics), X_3 [SCAT (Total)], X_{10} (Importance of Mathematics Grades).

Two factors were significantly correlated with achievement on the summative test but not on the postsummative test. These were X_8

(General Importance of Grades) and X_7 (General Self-Concept of Ability).

Factors X_4 (STEP) and X_1 [SCAT (Verbal)] bore no significant relationship (at the $\alpha = 0.05$ level) with achievement on either the summative or postsummative tests.

In every instance, the correlation coefficient of a student characteristic with performance on the summative test was higher than the correlation coefficient of the corresponding student characteristic with performance on the postsummative test.

Null Hypothesis 3(b):

The square of the multiple correlation coefficient between a combination of predictor variables (student characteristics) and the criterion achievement variable for a NNN student is not significantly different from zero.

The stepwise regression analysis outlined in Chapter IV resulted in regression equations for each of the summative and postsummative achievement tests. Information pertaining to the regression equations for the NNN students is summarized in Tables XXXIV and XXXV.

TABLE XXXIV

MULTIPLE REGRESSION EQUATION SIGNIFICANT
PREDICTORS (0.05 LEVEL) OF SUMMATIVE
TEST SCORES FOR NNN STUDENTS

Variable	Standard Weight	Weight	Standard Error
X_2	0.29	0.09	0.04
X_9	0.56	0.70	0.15

Constant = -5.93

Best Predictive Equation:

Y Summative = $-5.93 + 0.09X_2 + 0.70X_9$

Per cent Variance Accounted For = 57.16

Standard Error of Predicted Y = 5.03

X_2 = SCAT (Non-Verbal)

X_9 = Self-Concept of Mathematics Ability

Table XXXIV displays the variables that made a significant contribution in predicting the criterion variable (achievement on the summative test) for the NNN students. Thus, the best predictors of the performance of the NNN students on the summative test were:

X_2 : SCAT (Non-Verbal)

X_9 : Self-Concept of Mathematics Ability

The best equation for predictive purposes was:

$Y = -5.93 + 0.09X_2 + 0.70X_9$

Approximately 57 per cent of the variance of the criterion variable was accounted for by the predictors. The squared multiple correlation is significantly different from zero since the F_R value of 28.025 is greater than the critical value of 3.22 ($\alpha = 0.05$ level) for 2 and 42 degrees of freedom. Thus, Null Hypothesis 4(b) is rejected as far as the summative test achievement is concerned.

The regression equation would predict high summative test scores for NNN students with high SCAT (Non-Verbal) scores and high Self-Concept of Mathematics Ability scores.

TABLE XXXV

MULTIPLE REGRESSION EQUATION SIGNIFICANT PREDICTORS (0.05 LEVEL) OF POSTSUMMATIVE TEST SCORES FOR NNN STUDENTS

Variable	Standard Weight	Weight	Standard Error
X_1	-0.29	-0.09	0.04
X_2	0.42	0.13	0.04
X_5	0.48	3.72	1.06

Constant = 5.52

Best Predictive Equation:

Y Postsummative = $5.52 - 0.09X_1 + 0.13X_2 + 3.72X_5$

Per cent Variance Accounted For = 47.48

Standard Error of Predicted Y = 5.41

X_1 = SCAT (Verbal)
X_2 = SCAT (Non-Verbal)
X_5 = TEACHER EVALUATION OF ACHIEVEMENT

Table XXXV displays the variables that made a significant contribution in predicting the criterion variable (achievement on the postsummative test) for the NNN students. Thus, the best predictors of the performance of the NNN students on the postsummative test were:

X_1 : SCAT (Verbal)
X_2 : SCAT (Non-Verbal)
X_5 : TEACHER EVALUATION OF ACHIEVEMENT

The best equation for predictive purposes was:

$Y = 5.52 - 0.09X_1 + 0.13X_2 + 3.72X_5$

Approximately 47 per cent of the variance of the criterion variable was accounted for by the predictors. The squared multiple correlation is significantly different from zero since the F_R value of 12.353 is greater than the critical F value of 3.22 ($\alpha = 0.05$ level) for 3 and 41 degrees of freedom. Thus, Null Hypothesis 3(b) is rejected as far as the postsummative test achievement is concerned.

Null Hypothesis 3(b) is, therefore, rejected for each of the criterion variables (summative and postsummative test scores) although the combination of predictor variables was different in each instance.

QUESTION IV

To what extent does the mastery learning method of teaching mathematics affect the following student characteristics:

(a) student attitude toward mathematics?

(b) student general self-concept of ability?

(c) importance that the student attaches to grades in general?

(d) student self-concept of mathematics ability?

(e) importance that the student attaches to mathematics grades?

Following are the null hypotheses relative to Question IV. Each of the null hypothesis was analysed by means of the One-Way Analysis of Variance With Repeated Measures. The four groups are the MMM, NMM, NNM and NNN treatment groups.

Null Hypothesis 4(a):

There is no significant difference in the gains of the Attitude Toward Mathematics scores within each of the four groups of students.

Null Hypothesis 4(b):

There is no significant difference in the gains of the General Self-Concept of Ability scores within each of the four groups of students.

Null Hypothesis 4(c):

There is no significant difference in the gains of General Importance of Grades scores within each of the four groups of students.

Null Hypothesis 4(d):

There is no significant difference in the gains of the Self-Concept of Mathematics Ability scores within each of the four groups of students.

Null Hypothesis 4(e):

There is no significant difference in the gains of Importance of Mathematics Grades scores within each of the four groups of students.

Tables XXXVI, XXXVII, XXXVIII, XXXIX and XL contain details of the analyses of Null Hypotheses 4(a), 4(b), 4(c), 4(d) and 4(e), respectively.

TABLE XXXVI

DIFFERENCES OF MEANS OF ATTITUDE TOWARD MATHEMATICS TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS GROUPS

GROUP	N	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
MMM	48	87.13	83.44	- 3.69	0.075
NMM	26	86.00	75.12	-10.88	0.002
NNM	22	92.73	89.18	- 3.55	0.116
NNN	45	88.13	86.40	- 1.73	0.453

TABLE XXXVII

DIFFERENCES OF MEANS OF GENERAL SELF-CONCEPT OF ABILITY TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS GROUPS

GROUP	N	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
MMM	48	29.06	27.88	- 1.18	0.010
NMM	26	28.04	27.19	- 0.85	0.212
NNM	22	28.96	28.82	- 0.14	0.789
NNN	45	28.73	28.78	0.05	1.000

TABLE XXXVIII

DIFFERENCES OF MEANS OF GENERAL IMPORTANCE OF GRADES TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS GROUPS

GROUP	N	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
MMM	48	24.90	24.27	- 0.63	0.078
NMM	26	24.54	24.27	- 0.27	0.505
NNM	22	24.77	24.14	- 0.63	0.259
NNN	45	24.67	23.91	- 0.76	0.137

TABLE XXXIX

DIFFERENCES OF MEANS OF SELF-CONCEPT OF MATHEMATICS ABILITY TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS GROUPS

GROUP	N	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
MMM	48	21.10	20.58	- 0.52	0.122
NMM	26	20.96	20.08	- 0.88	0.026
NNM	22	21.36	20.73	- 0.63	0.370
NNN	45	20.76	20.56	- 0.20	0.597

TABLE XL

DIFFERENCES OF MEANS OF IMPORTANCE OF MATHEMATICS GRADES TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS GROUPS

GROUP	N	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
MMM	48	21.10	20.58	- 0.52	0.122
NMM	26	20.96	20.08	- 0.88	0.026
NNM	22	21.36	20.73	- 0.63	0.370
NNN	45	20.76	20.56	- 0.20	0.597

Tables XXXVI, XXXVII, XXXVIII, XXXIX and XL show that, apart from two exceptions (the NNN group increase of 0.05 between the pretest and posttest means of the General Self-Concept of Ability scores, and the NNM group increase of 0.68 between the pretest and posttest means of the Self-Concept of Mathematics Ability scores) all other groups experienced a decrease between the pretest and posttest means on the measures. Neither of the increases was significant; however, there were significant decreases between the pretest and posttest means as follows:

1. Attitude Toward Mathematics for the NMM Group;
2. General Self-Concept of Ability for the MMM Group;
3. Self-Concept of Mathematics Ability for the NMM Group; and
4. Importance of Mathematics Grades for the NMM Group.

As described in Chapter IV, each of the NMM and NNM groups comprised one class of students, while each of the MMM and NNN groups consisted of two classes of students. Each of the two teachers involved in the experiment taught one MMM class and one NNN class. In view of this arrangement, it was decided to look more closely at the performance of each of the six classes of students. Therefore, the same form of analysis was used but with respect to the six classes instead of the four groups of students (MMM, NMM, NNM and NNN treatment groups).

Tables XLI, XLII, XLIII, XLIV and XLV contain details of the analyses of differences of means within various classes.

TABLE XLI

DIFFERENCES OF MEANS OF ATTITUDE TOWARD MATHEMATICS TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS CLASSES

CLASS	N	TREATMENT	MEAN OF PRETEST SCORES $\overline{X}_1$	MEAN OF POSTTEST SCORES $\overline{X}_2$	DIFFERENCE $\overline{X}_2 - \overline{X}_1$	p
9A	26	MMM	86.00	75.12	-10.88	0.002
9B	25	MMM	86.92	81.72	- 5.20	0.043
9C	23	NNN	87.35	86.26	- 1.09	0.765
9D	22	NNM	92.73	89.18	- 3.55	0.116
9E	22	NNN	88.96	86.55	- 2.41	0.414
9F	23	NMM	87.35	85.30	- 2.05	0.545

TABLE XLII

DIFFERENCES OF MEANS OF GENERAL SELF-CONCEPT OF ABILITY TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS CLASSES

CLASS	N	TREATMENT	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
9A	26	NMM	28.04	27.19	- 0.85	0.212
9B	25	MMM	28.76	27.72	- 1.04	0.097
9C	23	NNN	29.22	29.44	0.22	0.734
9D	22	NNM	28.96	28.82	- 0.14	0.789
9E	22	NNN	28.23	28.09	- 0.14	0.823
9F	23	MMM	29.39	28.04	- 1.35	0.053

TABLE XLIII

DIFFERENCES OF MEANS OF GENERAL IMPORTANCE OF GRADES TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS CLASSES

CLASS	N	TREATMENT	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
9A	26	NMM	24.54	24.27	- 0.27	0.505
9B	25	MMM	24.20	23.28	- 0.92	0.110
9C	23	NNN	25.26	24.52	- 0.74	0.441
9D	22	NNM	24.77	24.14	- 0.63	0.259
9E	22	NNN	24.05	23.27	- 0.78	0.020
9F	23	MMM	25.65	25.35	- 0.30	0.461

TABLE XLIV

DIFFERENCES OF MEANS OF SELF-CONCEPT OF MATHEMATICS OF ABILITY TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS CLASSES

CLASS	N	TREATMENT	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
9A	26	NMM	25.69	25.19	- 0.50	0.374
9B	25	MMM	26.72	26.36	- 0.36	0.660
9C	23	NNN	26.78	26.61	- 0.17	0.854
9D	22	NNM	26.46	27.14	0.68	0.367
9E	22	NNN	26.50	25.59	- 0.91	0.128
9F	23	MMM	27.87	25.17	- 2.70	0.001

TABLE XLV

DIFFERENCES OF MEANS OF IMPORTANCE OF MATHEMATICS GRADES TEST SCORES (PRETEST AND POSTTEST) WITHIN VARIOUS CLASSES

CLASS	N	TREATMENT	MEAN OF PRETEST SCORES $\bar{X}_1$	MEAN OF POSTTEST SCORES $\bar{X}_2$	DIFFERENCE $\bar{X}_2 - \bar{X}_1$	p
9A	26	NMM	20.96	20.08	- 0.88	0.026
9B	25	MMM	20.40	19.96	- 0.44	0.319
9C	23	NNN	20.96	21.35	0.39	0.496
9D	22	NNM	21.36	20.73	- 0.63	0.370
9E	22	NNN	20.55	19.73	- 0.82	0.095
9F	23	MMM	21.87	21.26	- 0.61	0.249

Table XLVI, which follows, presents a concise summary of the results of Tables XLI, XLII, XLIII, XLIV and XLV.

TABLE XLVI

SUMMARY OF PROBABILITIES RESULTING FROM ONE-WAY ANALYSIS OF VARIANCE WITH REPEATED MEASURES

CLASS	TREAT-MENT	Attitude Toward Mathematics	General Self-Concept of Ability	General Importance of Grades	Self-Concept of Mathematics Ability	Importance of Mathematics Grades
9A	NMM	.002	.212	.505	.374	.026
9B	MMM	.043	.097	.110	.660	.319
9C	NNN	.765	.734	.441	.854	.496
9D	NNM	.116	.789	.259	.367	.370
9E	NNN	.414	.823	.020	.128	.095
9F	MMM	.545	.746	.461	.001	.249

From Table XLVI it is evident that, for the NMM group, the decreases between pretest and posttest means are still significant as far as Attitude Toward Mathematics and Importance of Mathematics Grades tests are concerned. However, where a significant decrease is shown for an MMM or NNN class there exists a considerable discrepancy compared with the other MMM or NNN class in terms of the probability (p).

In view of the wide discrepancies in the results of the analysis, it does not seem possible to reach valid conclusions concerning Question IV.

QUESTION V

Under mastery learning conditions, to what extent do students develop cooperation as opposed to competition in their learning?

This aspect of the experiment was undertaken as a case study in view of the numerous problems in establishing controls. For example:

1. It was not possible to observe each of the six classes in operation for more than one 20-minute period during each of the three sub-units. For one researcher to spend more time in observing may have endangered other more crucial aspects of the experiment.
2. Since the researcher concerned could not observe more than one class at one time, random selection of the periods of observation was impracticable.
3. Since the classes observed were conducted in their usual settings, some were observed in science laboratories and some in regular classrooms. The science laboratory setting may have encouraged students to work together, since students were seated in groups around tables. The regular classroom setting may have inhibited students' working together, since students were usually seated at individual desks arranged in columns.
4. Student attendance varied from period to period and also within periods as students were called away or were involved in other activities.
5. Occasionally, the teacher or the observer's colleague was unable to be present for a portion of, or for all of the

20-minute period of observation.

The procedures adopted were that, once housekeeping and routine matters (e.g., recording attendance) had been taken care of and instruction had started, the 20-minute period of observation commenced. Observation and recording ceased at the end of each 20-minute period.

The 20-minute period was divided into modules of one minute and a record kept of activity during each of the one-minute modules. The numbers and types of tutoring units (peer tutoring, teacher tutoring and researcher tutoring) were recorded for each one-minute module. Other activity (e.g., lectures or demonstrations by the teacher) was recorded in terms of one-minute modules.

Discussion of Question V will be restricted to the degree of tutoring during the 20-minute periods of observation.

Tables XLVII, XLVIII, XLIX, L, LI and LII indicate the number and type of tutoring units for classes 9A, 9B, 9C, 9D, 9E and 9F, respectively.

TABLE XLVII

TUTORING UNITS - CLASS 9A

Observation Period	Treatment	Tutoring Units				Comments
		Peer	Teacher	Researcher	Total	
1	N	1	14	9	24	Lecture for 13 minutes.
2	M	25	22	24	71	
3	M	37	24	18	79	
	Totals	63	60	51	174	

TABLE XLVIII

TUTORING UNITS - CLASS 9B

Observation Period	Treatment	Tutoring Units				Comments
		Peer	Teacher	Researcher	Total	
1	M	37	26	23	86	
2	M	22	36	26	84	
3	M	28	22	8	58	Researcher absent. Substitute present
	Totals	87	84	57	228	

TABLE XLIX

TUTORING UNITS - CLASS 9C

Observation Period	Treatment	Tutoring Units				Comments
		Peer	Teacher	Researcher	Total	
1	N	12	11	0	23	Lecture for 10 minutes. Researcher absent.
2	N	0	1	0	1	Lecture for 19 minutes.
3	N	25	30	0	55	Researcher absent.
	Totals	37	42	0	79	

TABLE L

TUTORING UNITS - CLASS 9D

Observation Period	Treatment	Tutoring Units				Comments
		Peer	Teacher	Researcher	Total	
1	N	16	5	6	27	Lecture for 9 minutes. Teacher & researcher absent for 4 minutes.
2	N	0	1	1	2	Lecture for 17 minutes.
3	M	34	16	15	65	Researcher absent. Substitute present.
	Totals	50	22	22	94	

TABLE LI

TUTORING UNITS - CLASS 9E

Observation Period	Treatment	Tutoring Units				Comments
		Peer	Teacher	Researcher	Total	
1	N	21	16	9	46	Lecture for 9 minutes.
2	N	20	18	0	38	Lecture for 7 minutes. Researcher absent.
3	N	8	25	0	33	Researcher absent.
	Totals	49	59	9	117	

TABLE LII

TUTORING UNITS - CLASS 9F

Observation Period	Treatment	Tutoring Units				Comments
		Peer	Teacher	Researcher	Total	
1	M	26	18	5	49	Teacher absent for 4 minutes. Researcher absent for 15 minutes.
2	M	36	16	18	70	
3	M	11	12	10	33	Researcher absent. Substitute present.
	Totals	73	46	33	152	

The 'Comments' sections of the above tables indicate when the teacher or Mr. Drost (researcher) was not available for tutoring. Also indicated are times when whole-class instruction took place. For example, Table L shows that:

1. During the first observation period, the teacher instructed the whole class for nine minutes. Later, the teacher and researcher were engaged in a discussion regarding the activity corner materials.
2. During the second observation period, the teacher instructed the whole group for 17 minutes.
3. During the third observation period, Mr. Drost was absent. Dr. A. T. Olson of the University of Alberta served as a substitute for Mr. Drost.

Table LIII, which follows, provides a summary of the tutoring for all six classes and permits some comparisons to be made.

TABLE LIII

TUTORING UNITS - SUMMARY FOR ALL CLASSES

CLASS	TREATMENT	TUTORING UNITS			
		PEER	TEACHER	RESEARCHER	TOTAL
9A	NMM	63	60	51	174
9B	MMM	87	84	57	228
9C	NNN	37	42	0	79
9D	NNM	50	22	22	94
9E	NNN	49	59	9	117
9F	MMM	73	46	33	152
	Total	359	313	172	844
	Mean	60	52	29	141

From Table LIII it is apparent that the largest number of tutorings (228) occurred during class 9B sessions (MMM). The lowest number of tutorings (79) occurred during class 9C (NNN) sessions. With regard to peer-tutoring, the MMM classes ranked highest (87 and 73) while the NNN classes ranked lowest (49 and 37). In view of the fact that a large proportion of the NNN class time was spent in whole-group instruction, valid comparisons cannot be made. Further, the several absences of a researcher during NNN sessions also affected the number of tutorings.

Thus, it is not possible from this study to provide a valid answer to Question V.

STUDENT OPINION REGARDING THE MASTERY LEARNING APPROACH

Following the writing of the Attitude Toward Mathematics and the Self-Concept of Ability and associated instruments posttests, the MMM students were asked to express their opinions regarding the mastery learning approach. They were also requested to suggest ways in which the method could be improved.

The following open-ended questions were asked:

1. How is this mathematics different from what you have had before?
2. How do you think we could improve on this type of mathematics?

The questionnaire may be found in Appendix E.

The responses to the question, "How is this mathematics different from what you have had before?", are summarized in Table LIV.

TABLE LIV

STUDENT OPINION REGARDING THE MASTERY LEARNING APPROACH

Response Number	RESPONSE	CRITERION ATTAINED		CRITERION NOT ATTAINED		ALL STUDENTS	
		Number of Responses	%	Number of Responses	%	Number of Responses	%
1	More individualized. Students received more individual assistance in overcoming problems.	15	75	15	54	30	63
2	More rushed. Greater pressures of time.	7	35	13	46	20	42
3	More interesting and enjoyable. Less boring.	8	40	5	18	13	27
4	Better organized. Students learned more easily and more quickly.	4	20	7	25	11	23
5	More difficult. More memorization required.	3	15	5	18	8	17
6	Too much spare time. Less homework.	3	15	1	4	4	8
7	Miscellaneous.	1	5	1	4	2	4

Table LIV shows that, of the 20 MMM students who attained criterion, 75 per cent were of the opinion that the mastery approach used, focused to a greater extent on the individual. Fifty-four per cent of the 28 students who did not attain criterion also felt that more attention was given to the individual student. Approximately two-thirds of the 48 students involved (63 per cent) expressed this opinion.

A large number of the students (42 per cent) felt they were more pressed for time. Several students thought that the instruction at the beginning of each subunit had been done too quickly. On the other hand, eight per cent of the students felt they had too much spare time in class and less homework than usual.

Approximately one-quarter of the students felt that the mastery method was better organized, more interesting and more enjoyable than their normal mode of instruction. They felt that they learned more easily and more quickly.

The responses to the question, "How do you think we could improve on this type of mathematics?", are summarized in Table LV.

TABLE LV

STUDENT SUGGESTIONS FOR IMPROVING THE MASTERY LEARNING APPROACH

Suggestion Number	SUGGESTION	CRITERION ATTAINED		CRITERION NOT ATTAINED		ALL STUDENTS	
		Number of Responses	%	Number of Responses	%	Number of Responses	%
1	Spend more time on initial instruction and/or throughout course, generally.	13	65	18	64	31	65
2	Provide greater opportunity for review.	4	20	3	11	7	15
3	Change formative test structure and testing procedures.	4	20	3	11	7	15
4	Omit answers from experience sheets.	5	25	1	4	6	13
5	Permit the more capable students to proceed without delays until whole unit is mastered.	3	15	1	4	4	8
6	Vary the degree of difficulty of work.	1	5	3	11	4	8
7	Change the location of the interest centre and increase the scope of its materials.	2	10	0	0	2	4
8	Miscellaneous.	1	5	1	4	2	4

Table LV shows that the most frequent suggestion for improvement was that more time should be spent on the initial instruction by the teacher and that the pace throughout should be reduced. Both the students who attained criterion and those who did not attain criterion were consistent in making this recommendation (65 per cent and 64 per cent, respectively).

Fifteen per cent of the total number of students felt that more opportunity for review should be provided.

Of the students who attained criterion, 25 per cent felt that answers should not be provided on the experience sheets. Apparently, the temptation to look up the answer before tackling the question was one that several students could not resist. However, only four per cent of the students who did not attain criterion held the same opinion.

From the comments and suggestions by the students, it is clear that they found their mastery lessons more consistent with the needs of the individual, more interesting and enjoyable, and organized in such a manner as to facilitate learning. Nevertheless, the majority felt that more time should have been spent during the initial instruction of each subunit and during the course itself.

CHAPTER VI

CONCLUSIONS AND IMPLICATIONS

I INTRODUCTION

For many years the public school has been the target for much criticism. Among the more vocal critics is Charles E. Silberman (1970) who has accused the school of "mindlessness", i.e., "the failure or refusal to think seriously about educational purpose, the reluctance to question established practice."

Central to this joint study was the intention of challenging at least one established and widespread belief - that only about one-third of our students are capable of learning well what is taught, a third will learn less well, and a third will fail or "get by" at best.

The purpose of the study was four-fold:

1. to demonstrate that most students can learn almost all of what is offered in school;
2. to devise a practicable strategy to achieve such a degree of learning;
3. to examine the effect of certain student characteristics on student achievement and, in turn, to examine the effect of achievement on these characteristics; and
4. to discover if student cooperation as opposed to competition develops under mastery learning conditions.

Dale Drost addressed himself to questions regarding differences in achievement between the mastery and non-mastery treatment groups; the relationship between student aptitude and attainment of mastery criterion; and the effect on the efficiency of the student's learning over the series of subunits into which the unit of mathematical content was divided.

This part of the joint study concerns itself mainly with the effect of a number of student characteristics on the students' learning and, in turn, the effect of the student's learning upon these student characteristics. The characteristics included both cognitive characteristics and affective characteristics (e.g., attitude, self-concept).

Implicit in the study was the belief that there is no one best method of instruction for all students. It was felt that certain teaching strategies were best for certain students in terms of their student characteristics. It was hoped to identify student characteristics important to the learning of a unit of algebra in a mastery learning situation and to compare these characteristics with those that were important to learning the same content in a non-mastery or "traditional" setting.

II THE STUDY

The mathematical content which was selected for the experiment was the segment of Grade 9 algebra which involves finding the products of polynomials and the factoring of polynomials.

The total content selected (the unit) was divided into three subunits each of which was further divided into 10 basic learning tasks of equal length and difficulty. The basic learning tasks were ordered hierarchically so that each task required mastery of the previous task.

Behavioral objectives were written for each learning task, and three activity experiences were prepared for each objective. A worksheet, four formative tests, and a review sheet, were designed for each of the three subunits. A summative test and postsummative test were constructed based on the content of the unit.

Both the mastery treatment and non-mastery treatment dealt with the same unit of content which was covered in the same amount of time. Essentially, the only difference was in the kind of instruction.

At the beginning of the first period of instruction on each subunit, each student (mastery and non-mastery treatment) received the objective sheets for the subunit.

In the mastery treatment, all objectives of the subunit were presented during the first one and one-half periods. Students received a worksheet on the objectives. Next, students wrote a formative test that identified weaknesses. The students were then directed to experiences to help overcome their difficulties. Each student was able to work at his own pace through a sequence of formative tests and corrective experiences until he had attained mastery on the subunit or until opportunity had expired. Opportunity included six class periods of time per subunit and three sets of experiences on each learning task. Students received a review sheet at the end of the subunit to enable them to review the work of the subunit before the summative test.

In the non-mastery treatment, the teachers followed their normal methods in teaching the objectives of each subunit. At the end of each subunit the students wrote a test on the subunit.

Six classes of Grade 9 students were involved. Two classes (MMM) experienced the mastery learning treatment for all three subunits and two classes (NNN) received the non-mastery treatment for all three subunits. A fifth class (NMM) received the non-mastery treatment for the first subunit and mastery treatment for the second and third subunits. A sixth class (NNM) experienced the non-mastery treatment for the first two subunits, and mastery treatment for the third subunit. All six classes wrote a summative test at the conclusion of the third subunit and wrote a postsummative test two weeks later.

Prior to the experiment all students completed the following instruments (pretests): Attitude Toward Mathematics Test, and Self-Concept of Ability and associated tests. Identical forms of these tests (posttests) were completed by the students following the writing of the summative test.

Following the writing of the posttests of the Attitude Toward Mathematics and Self-Concept of Ability and associated tests, the MMM students completed an opinionnaire regarding the mastery learning approach.

During the experiment, classroom observations were made in an effort to determine the degree to which students engaged in cooperation as opposed to competition in their learning activities. These observations were made during a 20-minute segment of one class period for each of the six classes and during each of the three subunits.

The two reports which were written on the joint study have certain features in common. Each discussed the rationale for and preparation of the materials used, the different treatments employed, and the design of the study. The companion report by Dale Drost also described the effect of the treatments on student achievement. His study tested various tenets of mastery learning theory including the relationship between student aptitude and achievement, and the relationship between time and achievement. This report has described the effect of a number of student characteristics on the learning of the students and, the effect of the students' learning upon these student characteristics. An attempt was also made to determine if the treatments affected the degree of cooperation and competition among the students.

The study was conducted at Sir George Simpson Junior High School in St. Albert. The sample consisted of all the Grade 9 students in the school (six classes; 153 students). Two teachers were involved. The content selected for the study was algebraic polynomials.

III DISCUSSION OF RESULTS

The results of this study were presented in detail in the previous chapter within the framework of the five questions posed in Chapter I. Before discussing the results it is appropriate that some comment be made regarding the influence that the materials and the design of the study had on the results.

The study sought to provide an example of a practicable mastery learning strategy whereby most of the students could learn almost all of the content presented to them. Now, it is essential that materials prepared for any mastery learning strategy include statements of what is to be learned; formative testing instruments which are both diagnostic and prescriptive; correctives which remedy deficiencies and help students to attain mastery; and summative testing instruments based on the statements of what was to be learned. The researchers believe that the materials prepared for this study possess these attributes.

The two most important characteristics of a mastery learning strategy are the effective use of feedback devices and corrective procedures, and the use of time as a variable. The diagnostic and prescriptive formative tests were used to advantage by many students in the experiment. However, despite concerted efforts by the researchers to convince the students that the formative tests would not be used for grading purposes, many students remained skeptical.

Mastery learning strategies seek to prove to students that they can master most of the material presented to them and, hence, achieve success in most of their educational experiences. In the course of this experiment it became evident that many students needed additional help on most or all of the tasks. It was unfortunate that many students continued to look upon the results of the formative tests as further confirmation of their inabilities and not as guides on the road to success. Numerous other students understood and accepted the role of the formative tests and used them to their benefit.

Time was used as a variable in the experiment; however, it was necessary to limit the class time for each subunit to six timetable periods per class. This limitation of time prevented some students from having the opportunity to take full advantage of all the appropriate materials and, consequently, were not sufficiently helped to attain mastery.

In the opinion of the researchers, many of the students who failed to attain mastery did not take full advantage of the materials available. Few students who continued to experience difficulties reached a point where they could work on the C-experiences. It is true that the limitation on class time was a detriment; however, it was obvious that some students were not willing to persevere during the necessary length of time, either during or out of class. Further, some students did not use the experiences to full advantage. They failed to give adequate attention to the supplementary explanations and examples before attempting the exercise portion of the experiences. Many of the students who made effective use of the experiences were able to attain the mastery criterion.

Neither the worksheets nor the review sheets were used effectively by many students. These materials were intended for student use outside regular class time and no check was made to ascertain if the students completed them. It appeared, however, that the highly motivated students made good use of the worksheets and review sheets whereas other less motivated students seemed to take little advantage of them.

The results discussed by Dale Drost in his report seem to indicate that both the mastery and non-mastery treatments devised for this study possessed properties of a mastery learning strategy. This outcome was expected for the mastery treatment but was not anticipated for the non-mastery treatment.

The non-mastery treatment differed from the teachers' normal techniques in some respects. For example, the non-mastery treatment included giving the students objective sheets and a test at the end of each subunit. This test was intended to be summative in that it contributed to the final grade of the student. The test, however, had a formative aspect in that it provided the students with an opportunity to diagnose their own weaknesses and to take remedial action before writing the summative test. The summative test was based on the instructional objectives of the mastery treatment. These departures from the teachers' regular practices resulted in the non-mastery treatment acquiring characteristics similar to those of a mastery learning strategy.

The results and conclusions which follow are summarized in the form of answers to the questions posed in Chapter I.

Question I. What differences exist in student characteristics between students who attain the achievement criterion and those who do not attain the achievement criterion

(a) in a mastery learning situation (MMM?

(b) in a non-mastery learning situation (NNN)?

Of the 48 MMM students, 20 students attained the mastery criterion of 80 per cent or higher on the summative test. Significant differences existed (at the $\alpha = 0.05$ level) between the distributions of certain scores for the MMM students who attained criterion and those MMM students who did not attain criterion on the summative test. The significant differences are as follows:

Variate	Probability (p)
SCAT (Verbal)	0.002
SCAT (Non-Verbal)	0.001
SCAT (Total)	0.0002
STEP	0.0003
TEACHER EVALUATION	0.0001
GENERAL SELF-CONCEPT OF ABILITY	0.033
SELF-CONCEPT OF MATHEMATICS ABILITY	0.046

Of the 45 NNN students, 10 students attained the mastery criterion of 80 per cent or higher on the summative test. Significant differences (at the $\alpha = 0.05$ level) existed between the distributions of certain scores for the NNN students who attained criterion and those NNN students who did not attain criterion on the summative test. The significant differences are as follows:

Variate	Probability (p)
SCAT (Non-Verbal)	0.001
SCAT (Total)	0.004
STEP	0.011
TEACHER EVALUATION	0.002
ATTITUDE TOWARD MATHEMATICS	0.004
GENERAL SELF-CONCEPT OF ABILITY	0.005
SELF-CONCEPT OF MATHEMATICS ABILITY	0.0003
IMPORTANCE OF MATHEMATICS GRADES	0.019

In comparing the differences which are significant between the students who attained the achievement criterion and those who did not attain criterion in both the mastery learning situation and the non-mastery learning situation, it is evident that several characteristics were significant in both situations.

The differences that were significant in both the MMM and the NNN situations are as follows:

SCAT (Non-Verbal)
SCAT (Total)
STEP
TEACHER EVALUATION
GENERAL SELF-CONCEPT OF ABILITY
SELF-CONCEPT OF MATHEMATICS ABILITY

The differences that were significant in only one situation are as follows:

SCAT (Verbal) which was significantly different only in the MMM situation, and

IMPORTANCE OF MATHEMATICS GRADES which was significantly different only in the NNN situation.

As a guard against the effects of positive correlations among the student characteristics the two-sample Hotelling T^2 test was used. In each of the MMM and NNN situations, the null hypothesis that the two samples (students who attained criterion versus students who did not attain criterion) arose from populations with a common mean vector, was rejected from a "global" standpoint. No single variables, or conceptually feasible linear combinations, were established as being significant (at the $\alpha = 0.05$ level) in the rejection of either of the null hypotheses. However, the 95 per cent confidence interval of the STEP/Teacher Evaluation of Achievement combination (lower bound - 1.49; upper bound 32.31) may have proven to be significant had the two samples been larger in which case the power of the analysis may have been greater.

Question II. What student characteristics can be used to predict mathematics achievement scores in a mastery learning setting?

The factor most highly correlated with achievement on the summative and postsummative tests was Teacher Evaluation of Achievement. This implies that students who received high scores in the last home report tended to have high scores on the summative and postsummative tests in the mastery learning situation.

The factor which bore the next higher correlation with achievement was STEP. The factors SCAT (Total), SCAT (Non-Verbal), General Self-Concept of Ability, SCAT (Verbal), Self-Concept of Mathematics Ability and Attitude Toward Mathematics bore a significant relationship with both criterion achievement tests.

Zero, or almost zero, correlation existed between achievement and General Importance of Grades and Importance of Mathematics Grades scores.

The factors which appeared in the regression equation as significant predictors (at the $\alpha = 0.05$ level) of summative test achievement were STEP (Total) and Teacher Evaluation of Achievement. These two variables accounted for approximately 65 per cent of the variance. Thus, the significant predictor variables for achievement on both criterion variables (summative and postsummative test scores) were STEP (Total) and Teacher Evaluation of Achievement. It would seem that students who have demonstrated high achievement by a standardized test and by the evaluation of their teacher, perform well under mastery learning conditions.

Question III. What student characteristics can be used to predict mathematics achievement scores in a non-mastery learning setting?

The factors most highly correlated with both criterion achievement tests were Self-Concept of Mathematics Ability and Teacher Evaluation of Achievement. This implies that students who possess a high self-concept of their ability in mathematics tend to have a high score on the summative and postsummative tests under the "regular" or non-mastery instructional mode. Further, students who received high scores in the last home report tended to have high scores on the summative and postsummative tests under the "regular" instructional mode. Other factors which were significantly related to both criterion achievement tests were SCAT (Non-Verbal), Attitude Toward Mathematics, SCAT (Total), and Importance of Mathematics Grades.

Two factors were significantly correlated with achievement on the summative test but not on the postsummative test. These were General Importance of Grades and General Self-Concept of Ability.

Neither STEP nor SCAT (Verbal) bore a significant relationship (at the $\alpha = 0.05$ level) with achievement on either the summative or postsummative tests.

In every instance, the correlation coefficient of a student characteristic with performance on the summative test was higher than the correlation coefficient of the corresponding student characteristic on the postsummative test.

The factors which appeared in the regression equation as significant predictors (at the $\alpha = 0.05$ level) of achievement on the summative test were SCAT (Non-Verbal) and Self-Concept of Mathematics Ability. These two variables accounted for 57 per cent of the variance.

The factors which appeared in the regression equation as significant predictors (at the $\alpha = 0.05$ level) of achievement on the postsummative test were SCAT (Verbal), SCAT (Non-Verbal) and Teacher Evaluation of Achievement.

Thus, SCAT (Non-Verbal) appeared in both regression equations. SCAT (Verbal), Self-Concept of Mathematics Ability and Teacher Evaluation of Achievement each appeared in one of the regression equations as predictors of achievement. These results are in contrast with the mastery learning situation where STEP (Total) and Teacher Evaluation of Achievement were the sole predictors of achievement in both regression equations.

Question IV. To what extent does the mastery-learning method of teaching mathematics affect the following student characteristics:

(a) student attitude toward mathematics?
(b) student general self-concept of ability?
(c) importance that the student attaches to grades in general?
(d) student self-concept of mathematics ability?
(e) importance that the student attaches to mathematics grades?

There were only two instances of an increase between the pretest and posttest means for the four groups which received the different treatments. The increases were; the NNN group increase of 0.05 between the pretest and posttest means of the General Self-Concept of Ability scores, and the NNM group increase of 0.68 between the pretest and posttest means of the Self-Concept of Mathematics Ability scores. All other groups experienced a decrease between the pretest and posttest means on the measures. Neither of the increases was significant; however, there were significant decreases (at the $\alpha = 0.05$ level) between the pretest and posttest means as follows:

1. Attitude Toward Mathematics for the NMM Group,
2. General Self-Concept of Ability for the MMM Group,
3. Self-Concept of Mathematics Ability for the MMM Group,
4. Importance of Mathematics Grades for the NMM Group.

As described in Chapter IV, each of the NMM and NNM groups comprised one class of students, while each of the MMM and NNN groups consisted of two classes of students. Each of the teachers who was involved in the experiment taught one MMM class and one NNN class. In view of this arrangement, it was decided to look more closely at the performance of each of the six classes.

Since the class which received the NMM treatment also constituted the NMM group, the decreases in means continued to be significant as far as Attitude Toward Mathematics and Importance of Mathematics Grades scores were concerned. However, where a significant decrease was found for one MMM or one NNN class there existed a considerable discrepancy compared with the other MMM or NNN class in terms of the probability (p).

In view of these wide discrepancies in the results, it did not seem proper to make valid conclusions concerning Question IV.

Question V. Under mastery learning conditions, to what extent do students develop cooperation as opposed to competition in their learning?

This aspect of the experiment was undertaken as a case study in view of the many problems which prevented adequate controls being established.

All six classes of students were observed for equivalent periods of time during each of the three subunits of instruction. From these observations it was possible to determine the number of peer tutorings, teacher tutorings and researcher tutorings which took place.

Although it was not possible to derive a statistically valid answer to Question V from the observation data, the information gathered does provide an interesting description of what transpired in the classroom during the periods of observation.

The largest total of peer, teacher and researcher tutoring (228) occurred during class 9B sessions (MMM). The lowest number of tutorings (79) occurred during class 9C (NNN) sessions. With regard to peer tutoring, the two MMM classes ranked highest (87 and 73) while the two NNN classes ranked lowest (49 and 37). A larger proportion of class time was spent on whole-group instruction in the non-mastery classes compared with the mastery classes.

Student Opinion Regarding the Mastery Learning Approach

In response to the open-ended question, "How is this mathematics different from what you have had before?", the following opinions were expressed.

Of the 20 MMM students who attained criterion, 75 per cent expressed the opinion that the mastery method focused to a greater extent on the individual than did the "traditional" method. Fifty-four per cent of the 28 students who did not attain criterion also felt that the individual student was given more attention. Approximately two-thirds of all the 48 MMM students involved expressed this opinion.

Many students (42 per cent) felt that they were more pressed for time. Several students stated that the instruction at the beginning of each subunit had been given too quickly. On the other hand, eight per cent of the students stated that they had too much spare time in class and less homework than usual.

Almost one-quarter of the students reported that the mastery method was better organized, more interesting and more enjoyable than the normal mode of instruction. They believed that they learned more easily and more quickly.

In response to the open-ended question, "How do you think we could improve on this type of mathematics?", the following suggestions were made by the MMM students.

The most frequent suggestion for improvement was that more time should be spent on the initial instruction by the teacher and that the pace throughout should be reduced. Both the students who attained criterion and those who did not attain criterion were consistent in making this recommendation (65 per cent and 64 per cent, respectively).

Fifteen per cent of the total number of students thought that more opportunity for review should be provided.

Of the students who attained criterion, 25 per cent stated that answers should not be provided on the experience sheets. It seemed that the temptation to look up the answer before tackling the question was one that several students could not resist. Yet, only four per cent of the students who did not attain criterion expressed the same opinion.

From their responses to the two open-ended questions, it is clear that the students found their mastery lessons more consistent with the needs of the individual, more interesting and enjoyable and organized in such a manner as to facilitate learning. Nevertheless, the majority felt that more time should have been spent during the initial instruction of each subunit and during the course itself.

IV LIMITATIONS OF THE STUDY

Several assumptions and limitations should be considered in interpreting the results of this study. In addition to the assumptions which were stated in Chapter I, the following limitations should be borne in mind.

The first limitation concerns the size of the samples. Complete data were obtained for only 48, 26, 22 and 45 students in the MMM, NMM, NNM and NNN groups, respectively.

The second limitation concerns the interaction effects of the four treatments. Although the extent of such interaction is difficult to establish, it was possible for students from the four groups to study together and share some of the materials prepared for the mastery learning treatment. Further, the non-mastery treatment was found to contain elements of a mastery learning strategy. D. Drost elaborated on this occurrence in his report.

Finally, the mastery learning strategy did provide the students with more practice in writing tests than did the non-mastery strategy, which may have had a bearing on performance on the achievement tests.

V IMPLICATIONS FOR INSTRUCTION

The main conclusion from this study is that there are student characteristics related to achievement in the mastery learning setting that are not so important in the non-mastery situation and vice versa. The implication from these results is that teachers should be cognizant of the characteristics of each student in order to determine which of the two methods of instruction may be best suited for the individual student.

The mastery learning treatment designed for the experiment proved to be quite practicable. Further, as D. Drost indicates in his report, most of the students who experienced the mastery learning strategy were able to learn most of the content which was presented to them. Further, the students felt that their lessons were more consistent with the needs of the individual, were more enjoyable and interesting, and were arranged in such a way as to facilitate their learning. Nevertheless, the majority of the students believed that more time should have been spent on the initial instruction and during the course itself.

It was not intended to establish statistically that the degree of cooperation among students became greater and competition decreased in a mastery learning environment. However, it appeared that the mastery learning treatment was conducive to individual students' receiving help with their problems. At any rate the mastery learning treatment allowed the teacher to have more individual contact than did the non-mastery treatment where contact was generally between the teacher and the group.

VI IMPLICATIONS FOR RESEARCH

It is recommended that further research related to mastery learning theory and strategies be conducted:

1. to determine how students possessing certain characteristics best learn mathematics and to devise strategies to ensure that these students attain mastery in their learning;
2. to determine precisely how the perceptions of students, regarding mathematics, the classroom situation, and school

in general, are affected by mastery learning approaches; and

3. to determine if the incidence of cooperation among students increases and competition decreases under mastery learning conditions in contrast to non-mastery conditions.

CHAPTER VII

RECENT DEVELOPMENTS IN MASTERY LEARNING THEORY AND PRACTICE

The Theoretical Basis

In 1963, John B. Carroll, in his Model of School Learning, provided a model for mastery learning which, some years later, inspired Bloom to derive his concept of mastery learning. Bloom (1973) stated that, "In setting time as the central variable in school learning, Carroll produced a major shift in our thinking about education and educational research."

According to Carroll, if all students were given the appropriate amount of time, i.e., the opportunity to learn, they could learn to the level of mastery. Further, the amount of time required could be greatly reduced if the quality of instruction were designed to meet the needs of the individual student.

In his work, Human Characteristics and School Learning, Bloom (1976) has made a major restatement of his theory that "what any person in the world can learn, almost all persons in the world can learn if provided with appropriate prior and current conditions of learning." Further, Bloom has organized the theory into a well defined framework that is supported by a substantial volume of research. Bloom's significant expansion of the Carroll model serves to explain individual differences in learning outcomes and to determine procedures whereby these individual differences may be altered or even eliminated.

Bloom identifies a small number of variables which will account for much of the variation in school learning. He contends that if three interdependent variables are given adequate attention, schools can come close to an error free system of schooling where most of the learners attain a high level of achievement, a relatively small amount of variation in levels of learning occurs, and the time required for learning shows a very small amount of variation.

The three interdependent variables which are central to Bloom's theory of school learning are:

1. the extent to which the student has already learned the basic prerequisites to the learning to be accomplished;
2. the extent to which the student is (or can be) motivated to engage in the learning process; and
3. the extent to which the instruction to be given is appropriate to the learner.

More specifically, Bloom's theory of school learning deals with the variables; student characteristics, instruction, and learning outcomes. Figure 4 shows in diagrammatical form the interrelationship of these variables.

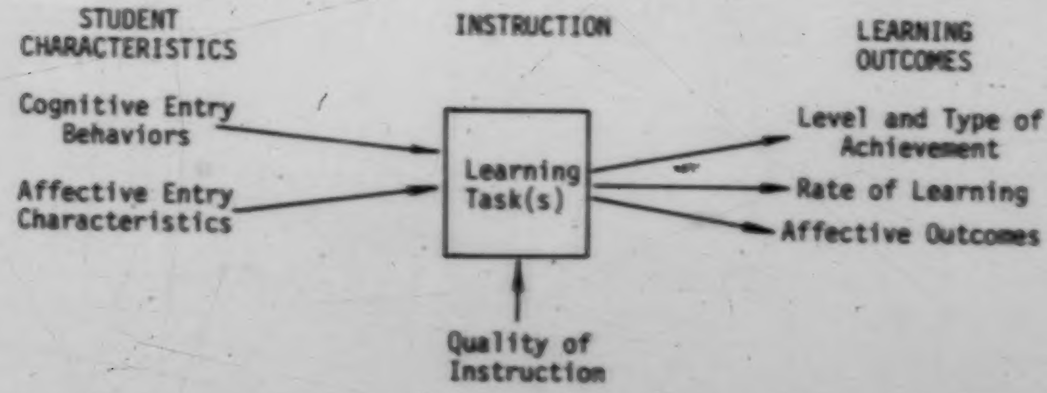

STUDENT CHARACTERISTICS
INSTRUCTION
LEARNING OUTCOMES
Cognitive Entry Behaviors
Affective Entry Characteristics
Learning Task(s)
Level and Type of Achievement
Rate of Learning
Affective Outcomes
Quality of Instruction

FIGURE 4

MAJOR VARIABLES IN THE THEORY OF SCHOOL LEARNING

According to Bloom, one of the student characteristics which is crucial in determining student learning is the Cognitive Entry Behaviours of the student, i.e., the prerequisite learning necessary for the learning task on which instruction is to be given. Another characteristic is the Affective Entry Characteristics, i.e., the motivation of the student to learn the new learning task. The instructional variable which is of greatest importance is the Quality of Instruction, i.e., the extent to which the cues, practice and reinforcement of the learning are appropriate to the needs of the learner.

The central thesis of Bloom's work is that variations in the three variables stated above will determine the nature of the learning outcomes which are; the Level and Type of Achievement, the Rate of Learning, and the Affective Characteristics of the Learner in relation to the learning task and to the self. If the student entry characteristics and the quality of instruction are favourable, then all the learning outcomes will be at a high or positive level and there should be little variation in the measures of the outcomes. On the other hand, if considerable variation exists among students in their entry characteristics and the quality of instruction is not optimal for the individual students, there should be wide variation in the learning outcomes.

Bloom sums the matter thus:

> The degree to which one or more of these variables is less than optimal will determine the level and type of learning achievement, the difficulties encountered in the learning process, the time and effort required to accomplish the learning that does take place, and the student's affect about the learning, the learning process, and the self.

With the publication of Bloom's book in 1976, the theory of mastery learning attracted a great deal of attention not only within the educational community, but also in other spheres.

Dolan (1978) has expressed the opinion that at a time when the school is being frequently and widely attacked as an effective institution, "Bloom's book must be considered one of the most optimistic views of the power of the school."

The Mastery Learning Strategy

The term "mastery learning" has been applied to a variety of recent instructional innovations (e.g., competency-based education and objectives-based education). One of the recently developed strategies which adheres closely to Bloom's theory is that developed by Block and Anderson (1975, 1977). The major thrust of this strategy focuses on two components; a commitment to criterion-referenced evaluation and a strong emphasis on feedback and correctives throughout the learning experience. The strategy possesses many advantages; for example, it is group-based which makes it practicable for regular classroom use. Further, apart from an initial investment of time in developing objectives. evaluation instruments, and corrective procedures, it does not require substantial expenditures of money for staff, equipment, or instructional materials. It is, therefore, a strategy that can be implemented in most schools within the resources that are currently available.

Block and Anderson have identified five aspects of the teacher's role: valuing, planning, managing, evaluating and revising. The aspect of valuing requires the teacher to make explicit the objectives of the instruction, to make value judgments regarding the content which has to be mastered by all students. The teacher is forced to consider, not only the criteria he will use to evaluate student achievement (the standard for mastery), but also to decide the sequence of instruction that best leads to mastery.

In the planning stage of instruction the teacher is required to develop a series of learning tasks which are subsets of the course. For each task, formative tests and corrective measures must be designed. The correctives may include alternative materials, peer and other tutoring, small group instruction, and any type of instruction which allows for differences in student characteristics. The final planning activity is the preparation of a summative test which is referenced to the objectives of the course.

The way in which correctives will be used presents the greatest challenge in the managing stage of instruction. The teacher has to decide on the amount of class time compared with out-of-class time to allow for correctives. Further, the teacher will have to find answers to other questions; for example, "Should a student who attains mastery quickly, be expected to tutor his peers who are still trying to achieve mastery, or should the student be encouraged to engage in activities to extend and enrich his original learning?" Teachers using the mastery learning approach have found that, in general, approximately 10 per cent more time is required for the corrective process, with more time needed in the early learning stage and less in the latter stage.

If the mastery strategy has been properly implemented, the summative test results should show that about 80 per cent of the students have attained a level of mastery that was previously attained by only 20 per cent. Further, the variation in student achievement should have decreased substantially.

Recent Research and Practice

The practice of mastery learning has not been implemented on a wide scale. Initially, most attempts at implementation were small experimental studies that involved highly sequential subject matter and limited samples of students. More recently, strategies have been implemented successfully in more typical school settings so that there is now ample proof of success in a variety of grade levels and areas of the curriculum.

The report by Smith and Katims (1977) on the Chicago Mastery Learning Reading Program (CMLRP) described efforts to implement a mastery learning system at the primary school level. The impact of CMLRP on teachers and students has provided considerable support in favour of mastery learning theory. Proponents of the CMLRP maintain that, since reading is one of the more difficult subject areas to adapt to the mastery learning model, the success of the Program provides proof that the model can work in any subject area. Jones, Gordon and Schechtman (1975) reported the success of a major experiment undertaken by the City Colleges of Chicago where the mastery approach was proven to be effective in a wide variety of course offerings, ranging from basic sciences to English composition. The success of these two programs is similar to the numerous small scale studies that have been conducted during the last decade.

Dolan (1978) summarized some of the more significant findings from experimental and field studies of mastery learning. Among the findings which he cites are the following:

1. Increased levels of achievement of students have been facilitated where students met the minimum standards of

participation. Higher levels of retention and future application of skills have been demonstrated.

2. General measurements of aptitude have failed to predict summative achievement within the mastery strategy, thus demonstrating that feedback and corrective strategies can overcome the specific history of the learner.
3. The investment of extra time early in the learning sequence is compensated by the more effective use of time at the later stages of learning. Further, this higher degree of active learning time is due to the student's internalization of instructional processing skills (e.g., the ability to evaluate, correct, and reward one's own learning). The student tends to have more control over his learning and becomes more self-reliant in the processing of instruction.
4. The learner tends to become more interested in the content being learned and feels more competent as a learner. Further, extended time under mastery conditions during the primary school years tends to have an impact on more general affective characteristics which parallel the dimensions of positive mental health. It is likely that these concerns, the motivation for future learning and the impact on personal adequacy, will have an effect comparable to the cognitive outcomes of the mastery strategy on the student's adaptation to future learning experiences.

Existing Problem Areas

Although the potential benefits of mastery learning have been confirmed by research and practice, criticism is still being raised regarding the utility of the mastery learning approach. Critics continue to cite content problems, student problems, and teacher problems. Some aspects of these problems will now be discussed.

There is a potential problem with the content to be mastered by all students. Decisions have to be made regarding content that is worth learning to the mastery level. Few would disagree that there is a common core of fundamental knowledge, "the basics," that would be very appropriate for mastery learning strategies. However, there is disagreement on other areas which are less basic such as the fine arts. It is very doubtful, also, if all content areas of the curriculum can be ordered sequentially and made explicit as is required by mastery approaches. Much work still remains to be done to assess the applicability of the strategy to higher level skills.

It has been claimed that mastery learning methods transfer the burden of the responsibility for learning from the student to the school, thereby reducing motivation. This claim has been disputed on the grounds that if a satisfactory balance is struck between the responsiblity of the student and the responsibility of the school, the student grows increasingly confident of his ability to learn and grows in self-reliance. Another complaint is that the goal of mastery for the majority can be attained only at the expense of the top 20 per cent of the group who would be held back while their slower classmates catch up. This complaint loses some of its force when the numerous positive effects are considered: the opportunity for the independent pursuit of enrichment activities, and the substantial cognitive and affective benefits of engaging in peer tutoring during the corrective phase of instruction.

Criticisms of teachers focus largely on implementation problems. Teachers have stated that it is not possible for one teacher to introduce adequate mastery strategies without the aid of other teachers within the same curriculum specialization. It has, in fact, become clear that the demands of developing objectives, diagnostic measures, and correctives are best carried forward by a team of teachers committed to the concept of mastery. Administrators have an important role in encouraging their teachers to introduce mastery systems, and to ensure that the necessary supportive networks, auxiliary services, and other resources are readily accessible to the teachers.

Future Prospects

In spite of many contrasting opinions it may be possible for some sort of consensus to be reached regarding the utility of mastery learning in the core of common learning which most agree is worth learning to mastery. Once it is established that "the basics" can be mastered, consideration can then be given to applications of the strategy in other areas of the curriculum. If efforts are concentrated on the early years of schooling, students who experience success in the basic skills will develop positive affective responses to subject content, feelings of competence and self-worth, and develop "learning-to-learn" techniques that are conducive to a positive history as a learner.

It is true that mastery learning is no cure-all. It will not always be possible to encounter success, to overcome in the case of some students, years of mindless neglect. There is no doubt that all teachers can benefit from the valuing, planning and management aspects of mastery learning instruction. Purkey (1970) has stated that very few students want to be failures at learning, just as very few teachers want to be failures at teaching. It may be that by applying mastery learning methods, the students and teachers may not only avoid failure but encounter success far beyond their present expectations. It is time to reject the traditional, established practice of sifting, sorting and labeling of students: it is time to concentrate on the development of the talent of students.

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APPENDIX A

OUTLINE OF UNIT III OF THE
GRADE NINE MATHEMATICS PROGRAM
OF SIR GEORGE SIMPSON SCHOOL

UNIT III: ALGEBRAIC EXPRESSIONS (POLYNOMIALS)

A. Powers of Numbers (Exponential Notation)

   1. Review

      (a) Powers of numbers, e.g., powers of 10, of 5, etc.

      (b) Base of powers

      (c) Exponents (indices)

      (d) Factors (prime, composite, common)

B. Integral Powers

   1. Positive integral powers

      (a) Introduce

         (i) $x^m \cdot x^n$

         (ii) $x^m \div x^n$

      (b) Do

         (i) $(x^m)^n$

         (ii) $(xy)^m$

   2. Zero powers

      (a) Review

         (i) Non-zero real numbers

         (ii) 0^0

   3. Negative integral powers

      (a) Review

         (i) $x^m \cdot x^n$

         (ii) $x^m \div x^n$

         (iii) $(x^m)^n$

         (iv) $(xy)^m$

   4. Computation with powers

5. Applications of base 10
    - (a) Expanded form, e.g., 400.74
    - (b) Scientific notation, e.g.,
        - (i) $5321 = 5.321 \times 10^3$
        - (ii) $5.32 = 5.32 \times 10^0$
        - (iii) $.0053 = 5.3 \times 10^{-3}$

C. Expressions

1. Language of algebra
    - (a) Variable
    - (b) Term, constant term
    - (c) Coefficients
    - (d) Factors of products
2. Operations involved in expressions (e.g., taking the additive inverse of a number)
3. Order of operations (e.g., $3 + 4 \times 5$, $(3 + 4) \times 5$)

D. Polynomials

1. Meaning of polynomial
    - (a) Monomial
    - (b) Binomial
    - (c) Trinomial
2. Degree of polynomial in one variable
    - (a) Degree of polynomial
    - (b) Linear polynomial
    - (c) Quadratic polynomial
3. Similar terms

4. Replacement of variables with constants (evaluation of expressions)

E. Operations with Polynomials

1. Addition of polynomials
    - (a) Review
        - (i) Similar terms
        - (ii) Properties needed in finding the sum
    - (b) Do addition of
        - (i) Two monomials, e.g., $(6e) + (13e)$
        - (ii) Monomial and binomial, e.g., $(5) + (7x + 8)$
        - (iii) Two binomials, e.g., $(7x + 5) + (9x - 8)$
          $(7x + 5) + (9x^2 - 8x)$
        - (iv) Other combinations of polynomials
2. Subtraction of polynomials
    - (a) Review--additive inverse
    - (b) Same as above for addition
3. Products of polynomials
    - (a) Two monomials
    - (b) Monomials and binomials
    - (c) Two binomials
        - (i) Any two binomials $(a + b)\ (c + d)$
        - (ii) Square of binomial $(a + b)^2$, $(a - b)^2$
        - (iii) Sum and difference $(a + b)\ (a - b) = a^2 - b^2$
4. Factoring of polynomials
    - (a) Removing a common factor which is a monomial, e.g., $2a^2 + 4ab$

(b) Removing a common factor which is a binomial, e.g., $2ab + 8a + 6b + 24$

(c) Difference of squares (as in 3(c), (iii))

(i) $x^2 - y^2$

(d) Trinomial squares

(i) $x^2 + 10x + 25 = (x + 5)^2$

(ii) $x^2 - 6x + 9 = (x - 3)^2$

(e) Trinomials

(i) Having 1 as the coefficient of the first term, e.g., $x^2 + 5x + 6$

(ii) Where a common factor can be removed first, e.g., $4x^2 + 20x + 24$

(iii) Having coefficient other than 1 for the first term, e.g., $2y^2 + 15y + 25$

5. Division of polynomials

(a) Review--reduction property

(b) Do division by

(i) The reduction property (factoring)

(ii) Long division

F. Rational Expressions

1. Rational expressions

(a) Meaning of rational expression

(b) Replacement of variables

(c) Meaningful replacements

(d) Reduction of quotients

2. Operations
    (a) Multiplication
        (i) Product property of quotients
        (ii) Examples, problems
    (b) Division
        (i) Reciprocal property of quotients
        (ii) Quotient property of quotients
        (iii) Examples, problems
    (c) Addition
        (i) Review of L.C.M.
        (ii) Sum property of quotients
        (iii) Examples, problems
    (d) Subtraction
        (i) Review of difference property
        (ii) Examples, problems

APPENDIX B

BASIC LEARNING TASK OBJECTIVES

BASIC LEARNING TASK OBJECTIVES
SUB-UNIT I

OBJECTIVE I.1

To find the product of two monomial expressions.

Examples

A. Find the product of $2x$ and $3y$.

B. Evaluate $4a^2b \cdot 7a^3b^3$.

Solutions

A. $2x \cdot 3y = 2 \cdot 3 \cdot x \cdot y$

$= 6xy$

B. $4a^2b \cdot 7a^3b^3 = 4 \cdot 7 \cdot a^2 \cdot a^3 \cdot b \cdot b^3$

$= 28a^5b^4$

OBJECTIVE I.2

To express a monomial expression as the product of two factors.

Examples

A. Find the missing factor $6x^2y^3 = (3xy) \cdot (\underline{\quad})$.

B. State two factors whose product is $10a^3b^2$.

Solutions

A. $6x^2y^3 = (3xy) \cdot (2xy^2)$

B. There are many possible solutions. For example:

(i) $2a^2b$, $5ab$
(ii) $10a^3$, b^2
(iii) $5ab^2$, $2a^2$

OBJECTIVE I.3

To find the product of a monomial and a binomial expression.

Examples

A. Find the product of $2x$ and $3y + 4x$.

B. Evaluate $4t^2(x - 5m)$.

Solutions

A. $2x \cdot (3y + 4x) = 2x \cdot 3y + 2x \cdot 4x$

$= 6xy + 8x^2$

B. $4t^2(x - 5m) = 4t^2 \cdot x - 4t^2 \cdot 5m$

$= 4t^2x - 20t^2m$

OBJECTIVE I.4

To express a binomial as the product of its greatest monomial factor and another binomial.

Examples

A. Express $3m^2 + 6mt$ as the product of its greatest monomial factor and another binomial.

B. Factor $4ab - 2b$.

Solutions

A. $3m^2 + 6mt = 3m \cdot m + 3m \cdot 2t$
$= 3m(m + 2t)$

B. $4ab - 2b = 2b \cdot 2a - 2b \cdot 1$
$= 2b(2a - 1)$

OBJECTIVE I.5

To find the product of a monomial and a trinomial expression.

Examples

A. Expand $2s(3t - 4st + 1)$.

B. Evaluate $4w^2(wx - 2y - 3w^3)$.

Solutions

A. $2s(3t - 4st + 1) = 2s \cdot 3t - 2s \cdot 4st + 2s \cdot 1$
$= 6st - 8s^2t + 2s$

B. $4w^2(wx - 2y - 3w^3) = 4w^2 \cdot wx - 4w^2 \cdot 2y - 4w^2 \cdot 3w^3$
$= 4w^3x - 8w^2y - 12w^5$

OBJECTIVE I.6

To express a trinomial as the product of its greatest monomial factor and another trinomial.

Examples

A. Factor $x^3 + 3x^2 - 4x$.

B. Find the greatest monomial factor of $6a^2y + 4ay^2 + 2a$ and write the trinomial as a product.

Solutions

A. $x^3 + 3x^2 - 4x = x \cdot x^2 + x \cdot 3x - x \cdot 4$
$= x(x^2 + 3x - 4)$

B. $6a^2y + 4ay^2 + 2a = 2a \cdot 3ay + 2a \cdot 2y^2 + 2a \cdot 1$
$= 2a(3ay + 2y^2 + 1)$

OBJECTIVE I.7

To express a binomial of the form $a(b + c) + d(b + c)$ as the product of two binomial factors.

Examples

A. Express $3q(y + 5) + 6(y + 5)$ as a product of two factors.

B. Factor $4x(2x - 1) - 3(2x - 1)$.

Solutions

A. $3q(y + 5) + 6(y + 5) = (3q + 6)(y + 5)$

B. $4x(2x - 1) - 3(2x - 1) = (4x - 3)(2x - 1)$

OBJECTIVE I.8

To find the product of two binomials of the form $(a + b)(c + d)$ where b and d are positive integers.

Examples

A. Find the product of $(2x + 5)$ and $(x + 3y)$.

B. Expand $(a + 2c)(2m + 3t)$.

Solutions

A.
$$\begin{aligned}(2x + 5)(x + 3y) &= 2x(x + 3y) + 5(x + 3y)\\ &= 2x \cdot x + 2x \cdot 3y + 5 \cdot x + 5 \cdot 3y\\ &= 2x^2 + 6xy + 5x + 15y\end{aligned}$$

B.
$$\begin{aligned}(a + 2c)(2m + 3t) &= a(2m + 3t) + 2c(2m + 3t)\\ &= 2am + 3at + 4cm + 6ct\end{aligned}$$

OBJECTIVE I.9

To find the product of two binomials of the form $(a + b)(c + d)$ where b and d are negative integers.

Examples

A. Expand $(x - 5)(m - 3)$.

B. Evaluate $(2a - 3b)(t - a)$.

Solutions

A.
$$\begin{aligned}(x - 5)(m - 3) &= x(m - 3) - 5(m - 3)\\ &= x \cdot m - x \cdot 3 - 5 \cdot m + 5 \cdot 3\\ &= xm - 3x - 5m + 15\end{aligned}$$

B.
$$\begin{aligned}(2a - 3b)(t - a) &= 2a(t - a) - 3b(t - a)\\ &= 2at - 2a^2 - 3bt + 3ab\end{aligned}$$

OBJECTIVE I.10

To find the product of two binomials of the form $(a + b)(c + d)$ where either b or d is a negative integer and the other is a positive integer.

Examples

A. What is the product of $(2q + 3t)$ and $(a - b)$.

B. Evaluate $(w - 6)(3 + 5y)$.

Solutions

A. $(2q + 3t)(a - b) = 2q(a - b) + 3t(a - b)$

$= 2qa - 2qb + 3ta - 3tb$

B. $(w - 6)(3 + 5y) = w(3 + 5y) - 6(3 + 5y)$

$= 3w + 5yw - 18 - 30y$

BASIC LEARNING TASK OBJECTIVES
SUB-UNIT II

OBJECTIVE II.1

To find the product of two binomials of the form $(x+a)(x+b)$.

Examples

A. Find the product of $(x+5)$ and $(x-4)$.

B. Expand $(m-6)(m-3)$.

Solutions

A. $(x+5)(x-4) = x(x-4) + 5(x-4)$
$= x^2 - 4x + 5x - 20$
$= x^2 + x - 20$

B. $(m-6)(m-3) = m(m-3) - 6(m-3)$
$= m^2 - 3m - 6m + 18$
$= m^2 - 9m + 18$

OBJECTIVE II.2

To find the product of two binomials of the form $(ax+b)(cx+d)$.

Examples

A. Expand $(2x+3)(x+1)$.

B. Evaluate $(3t-4)(2t+5)$.

Solutions

A. $(2x+3)(x+1) = 2x(x+1) + 3(x+1)$
$= 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$

B. $(3t-4)(2t+5) = 3t(2t+5) - 4(2t+5)$
$= 6t^2 + 15t - 8t - 20$
$= 6t^2 + 7t - 20$

OBJECTIVE II.3

To factor trinomials of the form $ax^2 + bx + c$ where $a=1$ and b and c are positive integers.

Examples

A. Factor $a^2 + 5a + 6$.

B. Factor $y^2 + 10y + 21$.

Solutions

A. Two numbers whose sum is 5 and product is $1 \cdot 6 = 6$ are 3 and 2.
$a^2 + 5a + 6 = a^2 + 3a + 2a + 6$
$= (a^2+3a) + (2a+6)$
$= a(a+3) + 2(a+3)$
$= (a+2)(a+3)$

B. Two numbers whose sum is 10 and product is $1 \cdot 21 = 21$ are 3 and 7.
$y^2 + 10y + 21 = y^2 + 3y + 7y + 21$
$= (y^2+3y) + (7y+21)$
$= y(y+3) + 7(y+3)$
$= (y+7)(y+3)$

OBJECTIVE II.4

To factor trinomials of the form $ax^2 + bx + c$, where b and c are positive integers.

Examples

A. Factor $6x^2 + 7x + 2$.

B. Factor $2t^2 + 13t + 15$.

Solutions

A. Two numbers whose sum is 7 and product is $6 \cdot 2 = 12$ are 3 and 4.

$$\begin{aligned} 6x^2 + 7x + 2 &= 6x^2 + 3x + 4x + 2 \\ &= (6x^2+3x) + (4x+2) \\ &= 3x(2x+1) + 2(2x+1) \\ &= (3x+2)(2x+1) \end{aligned}$$

B. Two numbers whose sum is 13 and product is $2 \cdot 15 = 30$ are 3 and 10.

$$\begin{aligned} 2t^2 + 13t + 15 &= 2t^2 + 3t + 10t + 15 \\ &= t(2t+3) + 5(2t+3) \\ &= (t+5)(2t+3) \end{aligned}$$

OBJECTIVE II.5

To factor trinomials of the form $ax^2 + bx + c$, where $a=1$ and b is a negative integer and c is a positive integer.

Examples

A. Factor $c^2 - 3c + 2$.

B. Factor $y^2 - 8y + 15$.

Solutions

A. Two numbers whose sum is -3 and product is $1 \cdot 2 = 2$ are -2 and -1.

$$\begin{aligned} c^2 - 3c + 2 &= c^2 - 2c - c + 2 \\ &= (c^2-2c) - (c-2) \\ &= c(c-2) - (c-2) \\ &= (c-1)(c-2) \end{aligned}$$

B. Two numbers whose sum is -8 and product is $1 \cdot 15 = 15$ are -5 and -3.

$$\begin{aligned} y^2 - 8y + 15 &= y^2 - 5y - 3y + 15 \\ &= (y^2-5y) - (3y-15) \\ &= y(y-5) - 3(y-5) \\ &= (y-3)(y-5) \end{aligned}$$

OBJECTIVE II.6

To factor trinomials of the form $ax^2 + bx + c$, where b is a negative integer and c is a positive integer.

Examples

A. Factor $5r^2 - 11r + 2$.

B. Factor $6x^2 - 17x + 5$.

Solutions

A. Two numbers whose sum is -11 and product is $5 \cdot 2 = 10$ are -10 and -1.

$$\begin{aligned} 5r^2 - 11r + 2 &= 5r^2 - 10r - r + 2 \\ &= (5r^2-10r) - (r-2) \\ &= 5r(r-2) - (r-2) \\ &= (5r-1)(r-2) \end{aligned}$$

B. Two numbers whose sum is -17 and product is $6 \cdot 5 = 30$ are -15 and -2.

$$\begin{aligned} 6x^2 - 17x + 5 &= 6x^2 - 15x - 2x + 5 \\ &= (6x^2-15x) - (2x-5) \\ &= 3x(2x-5) - (2x-5) \\ &= (3x-1)(2x-5) \end{aligned}$$

OBJECTIVE II.7

To factor trinomials of the form $ax^2 + bx + c$, where $a=1$ and b is a positive integer and c is a negative integer.

Examples

A. Factor $q^2 + 2q - 15$.

B. Factor $w^2 + 10w - 24$.

Solutions

A. Two numbers whose sum is 2 and product is $1 \cdot (-15) = (-15)$ are 5 and -3.

$$\begin{aligned} q^2 + 2q - 15 &= q^2 + 5q - 3q - 15 \\ &= (q^2+5q) - (3q+15) \\ &= q(q+5) - 3(q+5) \\ &= (q-3)(q+5) \end{aligned}$$

B. Two numbers whose sum is 10 and product is $1 \cdot (-24) = (-24)$ are -2 and 12.

$$\begin{aligned} w^2 + 10w - 24 &= w^2 - 2w + 12w - 24 \\ &= (w^2-2w) + (12w-24) \\ &= w(w-2) + 12(w-2) \\ &= (w+12)(w-2) \end{aligned}$$

OBJECTIVE II.8

To factor trinomials of the form $ax^2 + bx + c$, where b is a positive integer and c is a negative integer.

Examples

A. Factor $4x^2 + 7x - 2$.

B. Factor $6m^2 + 5m - 6$.

Solutions

A. Two numbers whose sum is 7 and product is $4 \cdot (-2) = (-8)$ are -1 and 8.

$$\begin{aligned} 4x^2 + 7x - 2 &= 4x^2 - x + 8x - 2 \\ &= (4x^2-x) + (8x-2) \\ &= x(4x-1) + 2(4x-1) \\ &= (x+2)(4x-1) \end{aligned}$$

B. Two numbers whose sum is 5 and product is $6 \cdot (-6) = (-36)$ are -4 and 9.

$$\begin{aligned} 6m^2 + 5m - 6 &= 6m^2 - 4m + 9m - 6 \\ &= (6m^2-4m) + (9m-6) \\ &= 2m(3m-2) + 3(3m-2) \\ &= (2m+3)(3m-2) \end{aligned}$$

OBJECTIVE II.9

To factor trinomials of the form $ax^2 + bx + c$, where $a=1$ and b and c are negative integers.

Examples

A. Factor $e^2 - 3e - 4$.

B. Factor $y^2 - 3y - 18$.

Solutions

A. Two numbers whose sum is -3 and product is $1 \cdot (-4) = (-4)$ are -4 and 1.

$$\begin{aligned} e^2 - 3e - 4 &= e^2 - 4e + e - 4 \\ &= (e^2-4e) + (e-4) \\ &= e(e-4) + (e-4) \\ &= (e+1)(e-4) \end{aligned}$$

B. Two numbers whose sum is -3 and product is $1 \cdot (-18) = (-18)$ are 3 and -6.

$$\begin{aligned} y^2 - 3y - 18 &= y^2 + 3y - 6y - 18 \\ &= (y^2+3y) - (6y+18) \\ &= y(y+3) - 6(y+3) \\ &= (y-6)(y+3) \end{aligned}$$

OBJECTIVE II.10

To factor trinomials of the form $ax^2 + bx + c$, where b and c are negative integers.

Examples

A. Factor $10t^2 - 11t - 6$. B. Factor $4a^2 - 8a - 21$.

Solutions

A. Two numbers whose sum is -11 and product is $10\cdot(-6) = (-60)$ are -15 and 4.

$$\begin{aligned} 10t^2 - 11t - 6 &= 10t^2 - 15t + 4t - 6 \\ &= (10t^2 - 15t) + (4t - 6) \\ &= 5t(2t-3) + 2(2t-3) \\ &= (5t+2)(2t-3) \end{aligned}$$

B. Two numbers whose sum is -8 and product is $4\cdot(-21) = (-84)$ are 6 and -14.

$$\begin{aligned} 4a^2 - 8a - 21 &= 4a^2 + 6a - 14a - 21 \\ &= (4a^2 + 6a) - (14a + 21) \\ &= 2a(2a+3) - 7(2a+3) \\ &= (2a-7)(2a+3) \end{aligned}$$

BASIC LEARNING TASK OBJECTIVES
SUB-UNIT III

OBJECTIVE III.1

To find the square of a binomial.

Examples

A. Square $(x+5)$. B. Evaluate $(2q-3)^2$.

Solutions

A. $(x+5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2$
$= x^2 + 10x + 25$

B. $(2q-3)^2 = (2q)^2 - 2 \cdot 2q \cdot 3 + (-3)^2$
$= 4q^2 - 12q + 9$

You might also use the methods of multiplying two binomials.

OBJECTIVE III.2

To factor a trinomial which is a perfect square.

Examples

A. Factor $y^2 - 6y + 9$. B. Factor $16m^2 + 24m + 9$.

Solutions

A. $y^2 - 6y + 9 = (y-3)^2$

B. $16m^2 + 24m + 9 = (4m+3)^2$

OBJECTIVE III.3

To find the product of the sum and difference of two terms.

Examples

A. Expand $(a+6)(a-6)$. B. Evaluate $(2x-3)(2x+3)$.

Solutions

A. $(a+6)(a-6) = a^2 - 6^2$
$= a^2 - 36$

B. $(2x-3)(2x+3) = (2x)^2 - (3)^2$
$= 4x^2 - 9$

OBJECTIVE III.4

To factor a binomial which is the difference of two squares.

Examples

A. Factor $s^2 - 4$. B. Factor $9p^2 - 16$.

Solutions

A. $s^2 - 4 = (s)^2 - (2)^2$
$= (s-2)(s+2)$

B. $9p^2 - 16 = (3p)^2 - (4)^2$
$= (3p-4)(3p+4)$

OBJECTIVE III.5

To factor a polynomial of 4 terms by grouping where no rearrangement of terms or sign alteration is necessary.

Examples

A. Factor $ax + 3x + ay + 3y$. B. Factor $4mt - 2mx + 6yt - 3yx$.

Solutions

A. $ax + 3x + ay + 3y = (ax+3x) + (ay+3y)$

$= x(a+3) + y(a+3)$

$= (x+y)(a+3)$

B. $4mt - 2mx + 6yt - 3yx = (4mt-2mx) + (6yt-3yx)$

$= 2m(2t-x) + 3y(2t-x)$

$= (2m+3y)(2t-x)$

OBJECTIVE III.6

To factor a polynomial of 4 terms by grouping and requiring sign alteration but not rearrangement of terms.

Examples

A. Factor $2t - 3tm - 4 + 6m$. B. Factor $4xy + 8xz - y - 2z$.

Solutions

A. $2t - 3tm - 4 + 6m = (2t-3tm) - (4-6m)$

$= t(2-3m) - 2(2-3m)$

$= (t-2)(2-3m)$

B. $4xy + 8xz - y - 2z = (4xy+8xz) - (y+2z)$

$= 4x(y+2z) - (y+2z)$

$= (4x-1)(y+2z)$

OBJECTIVE III.7

To remove a common factor from a trinomial and then factor the remaining trinomial of the form $ax^2 + bx + c$, $a=1$.

Examples

A. Factor $2x^2 + 10x - 12$. B. Factor $q^3 - 4q^2 + 3q$.

Solutions

A. $2x^2 + 10x - 12 = 2(x^2+5x-6)$

$= 2(x^2+6x-x-6)$

$= 2[x(x+6) - (x+6)]$

$= 2(x-1)(x+6)$

B. $q^3 - 4q^2 + 3q = q(q^2-4q+3)$

$= q(q-3)(q-1)$

OBJECTIVE III.8

To remove a common factor from a trinomial and then factor a remaining trinomial of the form $ax^2 + bx + c$.

Examples

A. Factor $8t^2 + 20t + 12$.   B. Factor $6m^2x - 5mx - 6x$.

Solutions

A. $8t^2 + 20t + 12 = 4(2t^2+5t+3)$
$= 4(2t+3)(t+1)$

B. $6m^2x - 5mx - 6x = x(6m^2-5m-6)$
$= x(3m+2)(2m-3)$

OBJECTIVE III.9

To remove a common factor from a trinomial and then factor the remaining trinomial as a perfect square.

Examples

A. Factor $3a^2 + 36a+108$.   B. Factor $4q^3m - 12q^2m + 9qm$.

Solutions

A. $3a^2 + 36a + 108 = 3(a^2+12a+36)$
$= 3(a+6)^2$

B. $4q^3m - 12q^2m + 9qm = qm(4q^2-12q+9)$
$= qm(2q-3)^2$

OBJECTIVE III.10

To remove a common factor from a binomial and then factor the remaining binomial as the difference of two squares.

Examples

A. Factor $8a^3 - 18a$.   B. Factor x^5-9x^3.

Solutions

A. $8a^3 - 18a = 2a(4a^2-9)$
$= 2a(2a-3)(2a+3)$

B. $x^5 - 9x^3 = x^3(x^2-9)$
$= x^3(x-3)(x+3)$

APPENDIX C

THE WORKSHEET, FORMATIVE TESTS,
EXPERIENCES AND REVIEW SHEET
USED IN SUBUNIT III

WORKSHEET III

THIS WORKSHEET IS TO GIVE YOU A BIT OF PRACTICE ON THE OBJECTIVES OF SUB-UNIT III. IF YOU LOOK AT THE EXAMPLES ON YOUR OBJECTIVE SHEETS YOU SHOULD BE ABLE TO DO EACH QUESTION. IF YOUR TEACHER DID NOT FINISH GOING OVER ALL THE OBJECTIVES IN CLASS, IT STILL WILL BE HELPFUL TO YOU TO DO ALL THE QUESTIONS.

REMEMBER: TAKE A LOOK AT THE EXAMPLES FIRST.

1. a. Square $(t-7)$.
   b. Evaluate $(2q+5)^2$.
2. a. Factor $m^2 + 14m + 49$.
   b. Factor $9m^2 - 30m + 25$.
3. a. Expand $(q+8)(q-8)$.
   b. Evaluate $(3r+2)(3r-2)$.
4. a. Factor $y^2 - 36$.
   b. Factor $25t^2 - 4$.
5. a. Factor $mr + nr + 3m + 3n$.
   b. Factor $5pq - 10pt + 2rq - 4rt$.
6. a. Factor $yz - 2z - 3y + 6$.
   b. Factor $4ab + 3bc - 8am - 6cm$.
7. a. Factor $qx^2 - 7qx + 12q$.
   b. Factor $3p^2 - 9p - 30$.
8. a. Factor $8x^2 + 14x + 6$.
   b. Factor $5rm^2 + 3rm - 2r$.
9. a. Factor $8y^2 - 16y + 8$.
   b. Factor $25w^2a + 20wa + 4a$.
10. a. Factor $a^2b - 4b$.
    b. Factor $50x^2 - 18$.

ANSWERS

1. $t^2 - 14t + 49$
   $4q^2 + 20q + 25$
2. $(m+7)^2$
   $(3m-5)^2$
3. $q^2 - 64$
   $9r^2 - 4$
4. $(y+6)(y-6)$
   $(5t+2)(5t-2)$
5. $(m+n)(r+3)$
   $(q-2t)(5p+2r)$
6. $(y-2)(z-3)$
   $(4a+3c)(b-2m)$
7. $q(x-4)(x-3)$
   $3(p-5)(p+2)$
8. $2(4x+3)(x+1)$
   $r(5m-2)(m+1)$
9. $8(y-1)^2$
   $a(5w+2)^2$
10. $b(a+2)(a-2)$
    $2(5x+3)(5x-3)$

FORMATIVE TEST III

NAME ______________________ DATE ________

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.

THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUB-UNIT III AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

		EXPERIENCE
1. Square $(y+4)$.	______________	III.1.A
2. Evaluate $(2t-1)^2$.	______________	
Factor the following as perfect squares:		
3. $a^2 - 8a + 16$	______________	III.2.A
4. $9w^2 + 12w + 4$	______________	
5. Expand $(x+5)(x-5)$.	______________	III.3.A
6. Evaluate $(2q+3)(2q-3)$.	______________	
Factor the following as the difference of squares:		
7. $m^2 - 16$	______________	III.4.A
8. $9b^2 - 25$	______________	
Group and then factor the following polynomials:		
9. $am + bm + 3a + 3b$	______________	III.5.A
10. $6xy - 4xz + 3y - 2z$	______________	
11. $wv - 2w - 5v + 10$	______________	III.6.A
12. $2mt + 12m - 3t - 18$	______________	
Factor the following by removing a common factor and then factoring the remaining trinomial:		
13. $3y^2 - 15y + 12$	______________	III.7.A
14. $q^2m + 3qm - 10m$	______________	
15. $9a^2 - 15a - 6$	______________	III.8.A
16. $6rw^2 + 7rw + 2r$	______________	
Factor the following by removing a common factor and then factoring the remaining trinomial as a perfect square:		
17. $2r^2 + 16r + 32$	______________	III.9.A
18. $9z^2m - 30zm + 25m$	______________	
Factor the following by removing a common factor and then factoring the remaining binomial as the difference of squares:		
19. $18x^2 - 8$	______________	III.10.A
20. $25tr^2 - 4t$	______________	

NOW TURN IN YOUR PAPER FOR MARKING

NOW THAT YOUR TEST HAS BEEN MARKED, YOU MAY FIND THAT YOU NEED SOME PRACTICE ON SOME OF THE OBJECTIVES. IF SO, YOU SHOULD PICK UP THE EXPERIENCE SHEETS CIRCLED ABOVE. THESE WILL HELP YOU TO OVERCOME YOUR DIFFICULTIES AND YOU MAY DO SOME AT HOME IF YOU WISH. IF YOU DID NOT MAKE ANY ERRORS YOU MAY WORK IN THE ACTIVITY CORNER OR HELP ANOTHER STUDENT WHO ASKS YOU FOR ASSISTANCE.

FORMATIVE TEST IIIA

NAME ______________________ DATE ________

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.
THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUB-UNIT III AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

		EXPERIENCE
1. Square $(2a+3)$.	________	III.1.B
2. Evaluate $(w-2)^2$.	________	
Factor the following as perfect squares:		
3. $t^2 + 6t + 9$	________	III.2.B
4. $16m^2 - 24m + 9$	________	
5. Expand $(a+4)(a-4)$.	________	III.3.B
6. Evaluate $(4x+3)(4x-3)$.	________	
Factor the following as the difference of squares:		
7. $c^2 - 4$	________	III.4.B
8. $16q^2 - 9$	________	
Group and then factor the following polynomials:		
9. $4xy + 2xz + 2y + z$	________	III.5.B
10. $ab - ac + 3xb + 3xc$	________	
11. $mt - 4m - 3t + 12$	________	III.6.B
12. $5c + 10x - ac - 2ax$	________	
Factor the following by removing a common factor and then factoring the remaining trinomial:		
13. $2z^2 + 10z + 12$	________	III.7.B
14. $px^2 - 2px - 8p$	________	
15. $8t^2 + 20t - 12$	________	III.8.B
16. $6pq^2 - 11pq + 3p$	________	
Factor the following by removing a common factor and then factoring the remaining trinomial as a perfect square:		
17. $ty^2 - 10ty + 25t$	________	III.9.B
18. $8m^2 + 24m + 18$	________	
Factor the following by removing a common factor and then factoring the remaining binomial as the difference of squares:		
19. $9xa^2 - 4x$	________	III.10.B
20. $3y^2 - 75$	________	

NOW TURN IN YOUR PAPER FOR MARKING

NOW THAT YOUR TEST HAS BEEN MARKED, YOU MAY FIND THAT YOU NEED SOME PRACTICE ON SOME OF THE OBJECTIVES. IF SO, YOU SHOULD PICK UP THE EXPERIENCE SHEETS CIRCLED ABOVE. THESE WILL HELP YOU TO OVERCOME YOUR DIFFICULTIES AND YOU MAY DO SOME AT HOME IF YOU WISH. IF YOU DID NOT MAKE ANY ERRORS YOU MAY WORK IN THE ACTIVITY CORNER OR HELP ANOTHER STUDENT WHO ASKS YOU FOR ASSISTANCE.

FORMATIVE TEST IIIB

NAME ______________________________ DATE __________

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.

THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUB-UNIT III AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

		EXPERIENCE
1. Square $(c-4)$.	______________	III.1.C
2. Evaluate $(2x+5)^2$.	______________	
Factor the following as perfect squares:		
3. $w^2 - 10w + 25$	______________	III.2.C
4. $4a^2 + 20a + 25$	______________	
5. Expand $(t+6)(t-6)$.	______________	III.3.C
6. Evaluate $(2r+5)(2r-5)$.	______________	
Factor the following as the difference of squares:		
7. $z^2 - 49$	______________	III.4.C
8. $4b^2 - 25$	______________	
Group and then factor the following polynomials:		
9. $3mn + 2my + 6n + 4y$	______________	III.5.C
10. $yt - zt + 4xy - 4xz$	______________	
11. $mk - m - 2k + 2$	______________	III.6.C
12. $2ax + 2ay - 3bx - 3by$	______________	
Factor the following by removing a common factor and then factoring the remaining trinomial:		
13. $qt^2 - 6qt + 5q$	______________	III.7.C
14. $4y^2 - 4y - 24$	______________	
15. $3r^2m + 7rm + 2m$	______________	III.8.C
16. $12k^2 - 14k - 6$	______________	
Factor the following by removing a common factor and then factoring the remaining trinomial as a perfect square:		
17. $3x^2 + 18x + 27$	______________	III.9.C
18. $4a^2y - 20ay + 25y$	______________	
Factor the following by removing a common factor and then factoring the remaining binomial as the difference of squares:		
19. $mr^2 - 49m$	______________	III.10.C
20. $12k^2 - 27$	______________	

NOW TURN IN YOUR PAPER FOR MARKING

NOW THAT YOUR TEST HAS BEEN MARKED, YOU MAY FIND THAT YOU NEED SOME PRACTICE ON SOME OF THE OBJECTIVES. IF SO, YOU SHOULD PICK UP THE EXPERIENCE SHEETS CIRCLED ABOVE. THESE WILL HELP YOU TO OVERCOME YOUR DIFFICULTIES AND YOU MAY DO SOME AT HOME IF YOU WISH. IF YOU DID NOT MAKE ANY ERRORS YOU MAY WORK IN THE ACTIVITY CORNER OR HELP ANOTHER STUDENT WHO ASKS YOU FOR ASSISTANCE

FORMATIVE TEST IIIC

NAME ______________________ DATE ________

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.

THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUB-UNIT III AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

1. Square $(3m-2)$. ______________
2. Evaluate $(a+5)^2$. ______________

Factor the following as perfect squares:

3. x^2+4x+4 ______________
4. $9y^2-6y+1$ ______________
5. Expand $(t+2)(t-2)$. ______________
6. Evaluate $(3m+2)(3m-2)$. ______________

Factor the following as the difference of squares:

7. b^2-25 ______________
8. $9z^2-4$ ______________

Group and then factor the following polynomials:

9. $5x+5y+xt+yt$ ______________
10. $3ab-2ac+6xb-4xc$ ______________
11. $tz-5t-3z+15$ ______________
12. $ax+ay-bx-by$ ______________

Factor the following by removing a common factor and then factoring the remaining trinomial:

13. $a^2b+4ab+3b$ ______________
14. $5m^2+10m-40$ ______________
15. $6k^2-21k+9$ ______________
16. $6r^2w+rw-2w$ ______________

Factor the following by removing a common factor and then factoring the remaining trinomial as a perfect square:

17. $qp^2-4qp+4q$ ______________
18. $27y^2+36y+12$ ______________

Factor the following by removing a common factor and then factoring the remaining binomial as the difference of squares:

19. $3x^2-27$ ______________
20. $4ab^2-25a$ ______________

NOW TURN IN YOUR PAPER FOR MARKING

EXPERIENCE III.1.A

NAME ____________________________ TIME STARTED TIME FINISHED

SQUARING A BINOMIAL IS THE SAME AS MULTIPLYING ANY TWO BINOMIALS EXCEPT WHEN YOU SQUARE, BOTH BINOMIALS ARE THE SAME.

$$\begin{aligned}(2x+3)^2 &= (2x+3)(2x+3) \longleftarrow \text{both the same}\\ &= 2x(2x+3) + 3(2x+3)\\ &= 4x^2 \boxed{+\ 6x + 6x} + 9 \longleftarrow \text{notice these are the same}\\ &= 4x^2 + 12x + 9\end{aligned}$$

IF YOU CAN GET TO THE LAST STEP IN YOUR HEAD WITHOUT MISTAKES -- GO AHEAD.

PLACE THE NUMBER OF THE CORRECT TRINOMIAL BESIDE ITS SQUARE:

______ $(t+5)^2$

______ $(t-3)^2$

______ $(2t+1)^2$

______ $(3t-2)^2$

______ $(2t+5)^2$

______ $(3t-1)^2$

9. $4t^2 + 4t + 1$
45. $9t^2 - 6t + 4$
1. $t^2 + 10t + 25$
16. $9t^2 - 12t + 4$
25. $4t^2 + 20t + 25$
38. $t^2 - 3t + 9$
36. $9t^2 - 6t + 1$
29. $4t^2 + 10t + 9$
4. $t^2 - 6t + 9$

DID YOU GET THE SQUARES OF THE FIRST SIX INTEGERS? IF YOU DID, YOU ARE ABSOLUTELY CORRECT!

EXPERIENCE III.1.B

NAME ______________________ TIME STARTED TIME FINISHED

$$(2x+3)^2 = \overset{\uparrow}{4x^2}\ \underbrace{+6x\ +6x}_{\uparrow} + \overset{\uparrow}{9} = 4x^2 + 12x + 9$$

square the first term

2 times the first term times the last term

square the last term

$(5x-2)^2$

Square the first term $\longrightarrow (5x)^2 = 25x^2$

2 times the first times the last $\rightarrow 2(5x)(-2) = -20x$

Square the last term $\longrightarrow (-2)^2 = 4$

$\longrightarrow$ Answer: $25x^2 - 20x + 4$

TRY $(x+3)^2$ .... DID YOU GET $x^2 + 6x + 9$?

NOW TRY THESE USING THE RULE:

(1) SQUARE THE FIRST

(2) 2 TIMES THE FIRST TIMES THE LAST

(3) SQUARE THE LAST

1. $(y+2)^2$
2. $(t-4)^2$
3. $(2a-1)^2$
4. $(3a+2)^2$
5. $(4p-5)^2$

$y^2 + 4y + 4$

$t^2 - 8t + 16$

$4a^2 - 4a + 1$

$9a^2 + 12a + 4$

$16p^2 - 40p + 25$

EXPERIENCE III.1.C

NAME ______________________ **TIME STARTED** **TIME FINISHED**

$$(\text{FIRST} + \text{LAST})^2 = \text{FIRST}^2 + 2\cdot\text{FIRST}\cdot\text{LAST} + \text{LAST}^2$$

$$(a + b)^2 = a^2 + 2\cdot a\cdot b + b^2$$

Eg. $(2w+3)^2$

$$\left.\begin{array}{l}\text{FIRST}^2 = (2w)^2 = 4w^2\\ 2\cdot\text{FIRST}\cdot\text{LAST} = 2\cdot 2w\cdot 3 = 12w\\ \text{LAST}^2 = (3)^2 = 9\end{array}\right\} 4w^2 + 12w + 9$$

$(p-6)^2$

$$\left.\begin{array}{l}\text{FIRST}^2 = (p)^2 = p^2\\ 2\ \text{FIRST LAST} = 2\cdot(p)\cdot(-6) = -12p\\ \text{LAST}^2 = (-6)^2 = 36\end{array}\right\} p^2 - 12p + 36$$

	FIRST	LAST	FIRST^2	2·FIRST·LAST	LAST^2	PRODUCT
$(x+3)^2$	x	3	x^2	$2\cdot x\cdot 3$	3^2	$x^2 + 6x + 9$
$(y-5)^2$	y	-5		$2\cdot y\cdot(-5)$		
$(2r+1)^2$	$2r$		$(2r)^2$			$4r^2 +$
$(3z-2)^2$		-2		$2\cdot(3z)\cdot(-2)$	$(-2)^2$	
$(t-4)^2$						
$(5m+2)^2$						

$x^2 + 6x + 9$

$y^2 - 10y + 25$

$4r^2 + 4r + 1$

$9z^2 - 12z + 4$

$t^2 - 8t + 16$

$25m^2 + 20m + 4$

EXPERIENCE III 2.A.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

Perhaps you made a careless error or two in the Formative Test and all you need is some more practice. Anyway, practice makes perfect!

Find an expression in the Column 2. which is equivalent to the perfect square trinomial in Column 1. Put your answers in the four by four square below. If you have answered the questions correctly you should find that the totals of the vertical columns, horizontal rows and diagonals are the same.

COLUMN 1.	COLUMN 1.
C. $x^2 + 6x + 9$	5. $(4x - 3)^2$
D. $x^2 - 6x + 9$	6. $(x - 3)^2$
E. $x^2 - 12x + 36$	7. $(2x - 5)^2$
F. $16x^2 - 8x + 1$	8. $(4x - 1)^2$
K. $4x^2 + 12x + 9$	13. $(2x + 3)^2$
L. $4x^2 - 20x + 25$	14. $(x - 6)^2$
M. $9x^2 + 30x + 25$	15. $(3x + 5)^2$
N. $16x^2 - 24x + 9$	16. $(x + 3)^2$

A 3	B 17	C	D
E	F	G 9	H 11
I 10	J 12	K	L
M	N	O 4	P 18

EXPERIENCE III. 2.B.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

A trinomial of the form $A^2 + 2AB + B^2$ is said to be a perfect square since it can be written as $(A + B)^2$.

Notice that $A^2 + 2AB + B^2$ consists of three terms.

(I) the square of a monomial A, plus

(II) the square of a monomial B, plus

(III) twice the product of A and B.

So, if we have to factor $x^2 - 6x + 9$ which is a perfect square we may write

$$x^2 - 6x + 9 = (x)^2 + 2(x)(-3) + (-3)^2$$
$$= (x - 3)^2$$

Now factor the following perfect squares.

1. $x^2 + 8x + 16 \quad = \quad (\quad + 4)^2$
2. $b^2 + 10b + 25 \quad = \quad (b + \quad)^2$
3. $49 - 14c + c^2 \quad = \quad (7 - \quad)^2$
4. $16p^2 + 40p + 25 \quad = \quad (4p + \quad)^2$
5. $x^2 - 12x + 36 \quad =$
6. $25p^2 + 10m + 1 \quad =$
7. $9y^2 - 30y + 25 \quad =$

Answers:

1. $(x + 4)^2$ 2. $(b + 5)^2$

3. $(7 - c)^2$ 4. $(4p + + 5)^2$

5. $(x - 6)^2$ 6. $(5p + 1)^2$

7. $(3y - 5)^2$

How did you get on? If you're having difficulty, get HELP!

EXPERIENCE III. 2.C.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

Hi there! Back for more? Stay with it! Let's look at the trinomial $b^2 + 6b + 9$. If we want to find its factors we need to find two numbers whose product is 9 and whose sum is 6. Did I hear you say 3 and 3?

$$\begin{aligned} \text{So, } b^2 + 6b + 9 &= b^2 + 3b + 3b + 9 \\ &= b(b + 3) + 3(b + 3) \\ &= (b + 3)(b + 3) \\ &= (b + 3)^2, \text{ a perfect square.} \end{aligned}$$

Now factor the following trinomial perfect squares.

1. $b^2 + 8b + 16$
2. $c^2 - 10c + 25$
3. $4p^2 + 4p + 1$
4. $9x^2 + 6x + 1$

Answers:

1. $(b + 4)^2$
2. $(c - 5)^2$
3. $(2p + 1)^2$
4. $(3x + 1)^2$

Check your answers. If you got all four correct I'll let you try the BIG ONE! Ready, go!

5. $25d^2 - 30d + 9$

Answer:

5. $(5d - 3)^2$

EXPERIENCE III. 3.A.

NAME: ____________________ TIME STARTED: TIME FINISHED:

Seems as though you did'nt get both questions 5 and 6 correct. Let's look at question 6 again. We will use the Distributive Property in the same way as we did way back in Objective I.8

$$(2q + 3)(2q - 3) = 2q(2q - 3) + 3(2q - 3)$$
$$= 4q^2 - 6q + 6q - 9$$
$$= 4q^2 - 9$$

Good grief! What happened to the two "inside terms" $-6q$ and $+ 6q$?

How would you like to try a few more examples?.... I just thought you would!

Find the product of the following:

1. $(x + 1)(x - 1)$
2. $(6 + b)(6 - b)$
3. $(3c + 2)(3c - 2)$
4. $(4 - 3x)(4 + 3x)$
5. $(11p - 1)(11p + 1)$
6. $(5t + 3)(5t - 3)$

Answers:

1. $x^2 - 1$ 2. $36 - b^2$
3. $9c^2 - 4$ 4. $16 - 9x^2$
5. $121p^2 - 1$ 6. $25t^2 - 9$

If you are still having trouble ASK SOMEONE FOR HELP------ IT'S FOR FREE!

EXPERIENCE III. 3.B.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

Hello again! Still having trouble finding the product of the sum and difference of two terms? Let's find the product of $(A + B)(A - B)$.

$(A + B)(A - B) = A(A - B) + B(A - B)$

$= A^2 - AB + BA - B^2$

Since $-AB + BA$ is zero,

$(A + B)(A - B) = A^2 + \text{zero} - B^2$

$= A^2 - B^2$

Now complete the following:

1. $(x + y)(x - y) = x(x - y) + y(x - y)$ Step 1.

$= x^2 -$ ___ $+$ ___ $- y^2$ Step 2.

$=$ ___ $-$ ___ Step 3.

2. $(2t + 3)(2t - 3) = 2t(2t - 3) + 3(2t - 3)$ Step 1.

$=$ ___ $-$ ___ $+$ ___ $-$ ___ Step 2.

$=$ ___ $-$ ___ Step 3.

Answers:

1. $x^2 - y^2$ 2. $4t^2 - 9$

Can you see how Step 2 might be eliminated? (Hint, two of the terms always add up to zero.)

Good luck in the next formative test!

EXPERIENCE III. 3.C.

NAME: ____________________ TIME STARTED: TIME FINISHED:

When we multiplied (A + B)(A - B) in Experience III.3.B. we found the product to be $A^2 - B^2$. Another way of saying this would be " The square of A minus the square of B "

In the same way,

$(x + y)(x - y)$ = (square of x) - (square of y)

= $x^2 - y^2$

Also, $(2t + 3)(2t - 3)$ = (square of 2t) - (square of 3)

= $(2t)^2 - (3)^2$

= $4t^2 - 9$.

Now complete the following:

1. $(c + d)(c - d)$ = (square of ____) - (square of ____)

= ____ - ____

2. $(2x + y)(2x - y)$ = (square of ____) - (square of ____)

= $(\_\_\_\_)^2 - (\_\_\_\_)^2$

= ____ - ____

3. $(3b + 2)(3b - 2)$ = $(\_\_\_\_)^2 - (\_\_\_\_)^2$

= ____ - ____

Answers:

1. $c^2 - d^2$ 2. $4x^2 - y^2$ 3. $9b^2 - 4$

If you didn't get all three correct - PLEASE ASK FOR HELP!

EXPERIENCE III.4.A

NAME ______________________ TIME STARTED TIME FINISHED

THE DIFFERENCE OF SQUARES HAS TWO MAIN CHARACTERISTICS. FIRST, THERE IS A DIFFERENCE AND SECOND, YOUR TWO TERMS MUST BOTH BE PERFECT SQUARES. FROM THE FOLLOWING LIST, SELECT THOSE WHICH WILL BY CALLED DIFFERENCE OF SQUARES, CIRCLE THE LETTER IN FRONT AND THEN FACTOR. CHECK YOUR OBJECTIVE SHEET TO SEE HOW TO FACTOR IF YOU HAVE FORGOTTEN. (OBJECTIVE III.4)

Q. $q^2 + 5 =$

C. $c^2 - 16 =$

A. $a^2 - 49 =$

P. $3p^2 - 16 =$

L. $4L^2 - 9 =$

M. $4m^2 - 7 =$

G. $25g^2 - 16 =$

T. $20t^2 - 1 =$

A. $9a^2 - 4 =$

R. $49r^2 - 1 =$

E. $16e^2 + 25 =$

Y. $64y^2 - 49 =$

DID YOU FIND OUT WHERE THE CALGARY STAMPEDE IS HELD?

C. $(c+4)(c-4)$
A. $(a+7)(a-7)$
L. $(2L+3)(2L-3)$
G. $(5g+4)(5g-4)$
A. $(3a+2)(3a-2)$
R. $(7r+1)(7r-1)$
Y. $(8y+7)(8y-7)$

EXPERIENCE III.4.B

NAME ______________________ **TIME STARTED** **TIME FINISHED**

MULTIPLY $(t+3)(t-3)$

$$\begin{aligned}(t+3)(t-3) &= t(t-3) + 3(t-3)\\ &= t^2 \underline{-3t + 3t} - 9 \\ &= t^2 - 9\end{aligned}$$

→ look, these add up to zero

FACTOR $t^2 - 9$

THIS IS REALLY THE SAME AS $t^2 + \underset{\overset{\uparrow}{\text{zero}}}{0t} - 9$ SO WE WANT TWO NUMBERS WHOSE SUM IS ZERO AND PRODUCT IS -9. THIS IS EASY! 3 AND -3

SO: $$\left.\begin{aligned}t^2 - 9 &= t^2 + 3t - 3t - 9\\ &= t(t+3) - 3(t+3)\\ &= (t+3)(t-3)\end{aligned}\right\} \text{just like our trinomials}$$

SINCE WE ARE DOING QUITE WELL, WE CAN GO FROM $t^2 - 9 = (t+3)(t-3)$ IF YOU REMEMBER THE SPECIAL TYPE OF DIFFERENCE OF SQUARES. IT SHORTENS YOUR WORK!

TRY THESE IN ONE STEP:

$m^2 - 4 =$

$y^2 - 49 =$

$4q^2 - 9 =$

$16x^2 - 25 =$

$36z^2 - 1 =$

GET THEM ALL?

$(m+2)(m-2)$

$(y+7)(y-7)$

$(2q+3)(2q-3)$

$(4x+5)(4x-5)$

$(6z+1)(6z-1)$

EXPERIENCE III.4.C

NAME ______________________ TIME STARTED TIME FINISHED

DIFFERENCE OF TWO SQUARES

$$\underset{\substack{\nearrow\\\text{perfect}\\\text{square}}}{\boxed{4x^2}}\ \overset{\substack{\text{difference}\\\downarrow}}{-}\ \underset{\substack{\nwarrow\\\text{perfect}\\\text{square}}}{\boxed{9}} = (2x)^2 - (3)^2 = (\underset{\substack{\uparrow\\\text{sum of}\\\text{terms}}}{2x+3})(\underset{\substack{\uparrow\\\text{difference}\\\text{of terms}}}{2x-3})$$

LOOK AT ANOTHER EXAMPLE:

$25m^2 - 16 = (5m)^2 - (4)^2 = (5m+4)(5m-4)$

STILL NEED HELP? ———— ASK FOR IT. IT IS FREE!!

TRY THESE:	ANSWERS
1. $t^2 - 25 =$	5. $(5z+6)(5t-6)$
2. $m^2 - 16 =$	4. $(3x+2)(3x-2)$
3. $4y^2 - 1 =$	3. $(2y+1)(2y-1)$
4. $9x^2 - 4 =$	2. $(m+4)(m-4)$
5. $25z^2 - 36 =$	1. $(t+5)(t-5)$

HOPE YOU WERE SUCCESSFUL!

EXPERIENCE III.5.A

NAME ______________________ TIME STARTED TIME FINISHED

YOU HAVE ALREADY FACTORED MANY POLYNOMIALS BY GROUPING. THE METHOD WE USED FOR FACTORING TRINOMIALS USED GROUPING.

$2x^2 + 11x + 12$

$= 2x^2 + 3x + 8x + 12$

$= (2x^2+3x) + (8x+12)$ ——— HERE IS YOUR GROUPING

$= x(2x+3) + 4(2x+3)$

$= (x+4)(2x+3)$

ALSO REMEMBER WAY BACK, WHEN WE DID QUESTIONS LIKE

$p(x+2) - 3(x+2)$ ——— ALREADY GROUPED

$= (p-3)(x+2)$

HERE THE GROUPING WAS ALREADY DONE FOR YOU.

LOOK AT $xy - 2x + 3y - 6$

Group $\longrightarrow$ $(xy-2x) + (3y-6)$

Common factor $\longrightarrow$ $x(y-2) + 3(y-2)$

Common factor $\longrightarrow$ $(x+3)(y-2)$

NOW TRY THESE:

1. $ab + a + b^2 + b$
2. $2xy - 2xz + 5y - 5z$
3. $4y - xy + 20 - 5x$
4. $mn + 2m + 3n + 6$

ANSWERS

$(a+b)(b+1)$

$(2x+5)(y-z)$

$(y+5)(4-x)$

$(m+3)(n+2)$

EXPERIENCE III.5.B

NAME ____________________ TIME STARTED TIME FINISHED

Jill: How do you factor a polynomial like $tg + 3t + 5g + 15$?

Bill: Look at the first two terms, $tg + 3t$; they have a common factor 't'.

Jill: Yes, I see that, and the last two terms have a common factor '5', but what good does that do?

Bill: Well, let's take out these common factors and see.

$$tg + 3t = t(g+3)$$
$$5g + 15 = 5(g+3)$$

Jill: Hey look, you got $g+3$ in both of them.

Bill: Right, and that is when the method works.

$$\begin{aligned} & tg + 3t + 5g + 15 \\ = & \ t(g+3) + 5(g+3) \\ = & \ (t+5)(g+3) \end{aligned}$$

Jill: Will this always work?

Bill: Only in cases where we can take out a common factor from the first two terms and another common factor from the last two terms and then have the same factor, like '$g+3$' appear. If all this does not happen, we will have to extend our method a bit.

Jill: That's O.K. because in all the questions from this objective, our method will work.

FACTOR THESE:

1. $xy - 3x + 5y - 15$
2. $mn + 2n + 3ym + 6y$
3. $ab^2 - cb^2 + 5a - 5c$
4. $10 + 5p + 2q + pq$
5. $4ab - 6ac + 6bx - 9cx$

ANSWERS

1. $(x+5)(y-3)$
2. $(n+3y)(m+2)$
3. $(b^2+5)(a-c)$
4. $(5+q)(2+p)$
5. $(2a+3x)(2b-3c)$

EXPERIENCE III.5.C

NAME ______________________________ TIME STARTED TIME FINISHED

EXAMPLE

FACTOR	$3ax - 5bx + 3ay - 5by$	
Step 1.	$(3ax-5bx) + (3ay-5by)$	Group -- Check signs
Step 2.	$x(3a-5b) + y(3a-5b)$	Take a common factor from each group
Step 3.	$(x+y)(3a-5b)$	Take out the binomial common factor

ALWAYS USE THESE THREE STEPS FOR FACTORING A POLYNOMIAL OF 4 TERMS BY GROUPING.

1. GROUP THE TERMS IN EQUAL GROUPS (WATCH YOUR SIGNS).
2. REMOVE A COMMON FACTOR FROM EACH GROUP.
3. REMOVE A COMMON FACTOR FROM THE POLYNOMIAL.

TRY THESE:

$xy - y + 2x - 2 = (xy-y) + (2x-2) = y(x-1) + 2(x-1) =$ ____________________

$2pq + 4p + 3q + 6 = (2pq+4p) + (3q+6) = 2p($______$) + 3($______$) =$ ____________________

$a^2 - 4ab + 2a - 8b = (a^2-4ab) + (2a-8b) =$ ___$(a-4b) +$ ___$(a-4b) =$ ____________________

$3mn + 3mc + 2n + 2c = (3mn+3mc) + (2n+2c) =$ $=$ ____________________

$rt - t^2 + r - t =$ $=$ $=$ ____________________

$6c + 2c^2 + 9a + 3ac =$ $=$ $=$ ____________________

ANSWERS

$(y+2)(x-1)$
$(2p+3)(q+2)$
$(a+2)(a-4b)$
$(3m+2)(n+c)$
$(t+1)(r-t)$
$(2c+3a)(3+c)$

EXPERIENCE III.6.A.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

Questions 11 and 12 on Formative Test III involved factoring a polynomial of 4 terms by grouping and SIGN ALTERATION. Let's look at number 11.

$$wv - 2w - 5v + 10$$

The first two terms have a common factor w so we can write, $wv - 2w$ as $w(v - 2)$. Can we factor $-5v + 10$ so that one of its factors is $(v - 2)$? You bet!

$$\begin{aligned} -5v + 10 &= -(5v - 10) \\ &= -5(v - 2) \end{aligned}$$

NOTICE THE SIGN CHANGE

So, our original polynomial can be factored like this:

$$\begin{aligned} wv - 2w - 5v + 10 &= w(v - 2) - 5(v - 2) \\ &= (w - 5)(v - 2) \end{aligned}$$

Now try to factor these:

1. $bc - 3c - 2b + 6$
2. $3xy + 9xz - y - 3z$
3. $4mt - 2mx - 6ty + 3xy$
4. $15pr - 5ps - 3qr + qs$

Answers:

1. $(c - 2)(b - 3)$
2. $(3x - 1)(y + 3z)$
3. $(2m - 3y)(2t - x)$
4. $(5p - q)(3r - s)$

EXPERIENCE III.6.B.

NAME: ____________________ TIME STARTED: TIME FINISHED:

I wonder if you are making mistakes with the SIGNS? Let's find out.

Answer true or false to the following.

1. $2df + 2dk = 2d(f + k)$ ____
2. $-3f - 3k = -3(f + k)$ ____
3. $-4k - 4 = -4(k - 1)$ ____
4. $3df - 3dk = 3d(f - k)$ ____
5. $-2f + 2k = -2(f - k)$ ____
6. $-2t + 6 = -2(t + 3)$ ____

Answers: 1. T. 2. T. 3. F. 4. T
5. T. 6. F.

Now use some of the above information to factor the following by grouping and sign alteration.

7. $2df + 2dk - 3f - 3k$ ____________________
8. $3df - 3dk - 2f + 2k$ ____________________

Answers: 7. $(2d - 3)(f + k)$
8. $(3d - 2)(f - k)$

GET HELP if you didn't get questions 7 and 8 correct!

EXPERIENCE III.6.C.

NAME: ____________________ TIME STARTED: TIME FINISHED:

Please look over Experience III.6.A. Do you understand how we grouped and then factored the polynomial wv - 2w - 5v + 10? Of course, we had to change some SIGNS which can be tricky if we don't take care. If you don't fully understand how the polynomial was factored please ASK SOMEONE TO HELP YOU.

Now use the same method to factor, st - 2s - 4t + 8

____

Did you get (s - 4)(t - 2)?

Now I would like you to factor the following polynomials. Each will involve GROUPING and

SIGN ALTERATION.

Good Luck!

1. bc + bd - fc - fd
2. 3bc + 6bd - 2c - 4d
3. 2xz - 6x - yz + 3y

Answers:

1. (b - f)(c + d)
2. (3b - 2)(c + 2d)
3. (2x - y)(z - 3)

EXPERIENCE III.7.A.

NAME: ____________________ TIME STARTED: TIME FINISHED:

Objective III.7 involves TWO STEPS.

STEP 1. Remove a common factor.

STEP 2. Factor the remaining trinomial.

Example: Factor $3x^2 + 15x + 18$

STEP 1. Remove the common factor 3

So we have,

$$3x^2 + 15x + 18 = 3(x^2 + 5x + 6)$$

STEP 2. Factor the trinomial $x^2 + 5x + 6$

$$\begin{aligned} x^2 + 5x + 6 &= x^2 + 2x + 3x + 6 \\ &= x(x + 2) + 3(x + 2) \\ &= (x + 3)(x + 2) \end{aligned}$$

So, $3x^2 + 15x + 18 = 3(x + 3)(x + 2)$

Now factor these polynomials:

1. $3b^2 + 15b + 18$
2. $2x^2 + 8x - 10$
3. $p^3 + 5p^2 + 6p$
4. $3c^2 - 6c - 24$
5. $d^3 - 7d^2 + 10d$

Answers:

1. $3(b + 3)(b + 2)$
2. $2(x + 5)(x - 1)$
3. $p(p + 3)(p + 2)$
4. $3(c - 4)(c + 2)$
5. $d(d - 5)(d - 2)$

EXPERIENCE III.7.B.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

Still having difficulty? Let's try to factor $4t^2 + 4t - 24$. What common factor can we take out? ______________

What trinomial are we left with? ______________

What are the factors of this trinomial? ______________

Now write down the factors of $4t^2 + 4t - 24$ ______________

Check your answers. If you did not get all correct ASK FOR HELP!

Now, use the same procedure to factor $3t^2 - 15t + 12$. Good luck!

Now, try to factor $km^2 - 6km + 5k$

Answers:

4

$t^2 + t - 6$

$(t + 3)(t - 2)$

$4(t + 3)(t - 2)$

$3(t - 4)(t - 1)$

$k(m - 1)(m - 5)$

EXPERIENCE III.7. C.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

Congratulations on staying with it! Remember, "A quitter never wins, and a winner never quits" !

I wonder if you did Experience II.9.C? Let's look back and use it to help factor $2x^2 + 10x + 12$. If we take out the common factor 2 we get $2(x^2 + 5x + 6)$. Now, if you look at Example 1. you will see how $x^2 + 5x + 6$ is factored. So, factors for $2x^2 + 10x + 12$ are $2(x + 3)(x + 2)$.

Now try to factor:

1. $3x^2 - 18x + 24$
2. $4x^2 + 16x - 20$
3. $5x^2 - 10x - 40$

Hint: Take out a common factor and then see if Examples 2, 3 and 4 help you out.

Check your answers. If you didn't get all correct ASK FOR HELP!

Answers:

1. $3(x - 4)(x - 2)$
2. $4(x + 5)(x - 1)$
3. $5(x - 4)(x + 2)$

EXPERIENCE III.8.A

NAME ______________________ TIME STARTED TIME FINISHED

THIS OBJECTIVE USES SOME OF THE PREVIOUS OBJECTIVES YOU HAVE LEARNED. FIRST IN OBJECTIVE I.6, YOU REMOVED A COMMON FACTOR FROM A TRINOMIAL AND IN OBJECTIVE II.4, II.6, II.8 AND II.10 YOU FACTORED TRINOMIALS OF THE FORM $ax^2 + bx + c$ WHERE $a \neq 1$. IN THIS OBJECTIVE YOU DO BOTH OF THESE.

EXAMPLE: Factor $10r^2 - 22r + 4$

$$= 2(5r^2 - 11r + 2) \longleftarrow \text{remove a common factor}$$

If you have forgotten how to factor trinomials you should refer back to the objectives mentioned above.

$$\left.\begin{aligned} &= 2\{5r^2 - 10r - r + 2\} \\ &= 2\{5r(r-2) - (r-2)\} \\ &= 2(5r-1)(r-2) \end{aligned}\right\} \longleftarrow \text{factoring a trinomial}$$

IF YOU THINK YOU NEED HELP ——— ASK. IF NOT, TRY THESE.

REMEMBER: REMOVE THE COMMON FACTOR FIRST!

Factor:

1. $2ax^2 + 5ax + 2a$
2. $9m^2 - 12m - 12$
3. $12bx^2 + 21bx - 6b$
4. $6ryp^2 - 11ryp + 3ry$

ANSWERS

$a(2x+1)(x+2)$

$3(3m+2)(m-2)$

$3b(4x-1)(x+2)$

$ry(3p-1)(2p-3)$

EXPERIENCE III.8.B

NAME ____________________ TIME STARTED TIME FINISHED

TWO STEPS

#1. REMOVE THE COMMON FACTOR FROM EACH OF THE FOLLOWING:

a. $2em^2 + 5em + 3e =$

b. $10ab^2 - 14ab - 12a =$

c. $12st^2 - 24st + 9s =$

d. $24z^2y + 20zy - 4y =$

DID YOUR COMMON FACTORS COME OUT TO BE:

$\underline{e}$ ——— $2\underline{a}$ ——— $3\underline{s}$ ——— $4\underline{y}$

#2. NOW LET'S FACTOR YOUR TRINOMIALS. THE QUESTIONS IN PART 2 ARE THE ANSWERS IN PART 1.

a. $e(2m^2 + 5m + 3) = e(\quad)(\quad)$

b. $2a(5b^2 - 7b - 6) =$

c. $3s(4t^2 - 8t + 3) =$

d. $4y(6z^2 + 5z - 1) =$

IF YOU ARE HAVING TROUBLE WITH THESE TRINOMIALS, LOOK BACK AT OBJECTIVES II.4, II.6, II.8 AND II.10. PERHAPS A FRIEND CAN HELP YOU IF YOU ARE REALLY STUCK.

$e(2m+3)(m+1)$ $2a(5b+3)(b-2)$ $3s(2t-1)(2t-3)$ $4y(6z-1)(z+1)$

EXPERIENCE III.8.C

NAME ______________ TIME STARTED TIME FINISHED

STILL HAVING TROUBLES? HERE IS ONE EXAMPLE:

FACTOR COMPLETELY: $12ab^2 - 10ab - 12a$

Remove common factor ⟶ $= 2a(6b^2 \underline{-5b} - 6)$

Find 2 numbers whose product is (6)(-6)= -36 and sum is -5. Remember this. numbers are (-9) and 4. ⟶ $= 2a(6b^2 \underline{-9b + 4b} - 6)$

Grouping ⟶ $= 2a\left\{(6b^2-9b) + (4b-6)\right\}$

Remove a common factor from each group ⟶ $= 2a\left\{3b(2b-3) + 2(2b-3)\right\}$

Remove common factor (2b-3) ⟶ $= 2a(3b+2)(2b-3)$

IF YOU STILL ARE HAVING TROUBLE --- GET SOME HELP!

NOW TRY THESE:

Polynomial	Common Factor	Trinomial	Factors of Trinomial	Factors of Polynomial
$10qm^2 - 12qm + 2q$	$2q$	$5m^2 - 6m + 1$	$(5m-1)(m-1)$	$2q(5m-1)(m-1)$
$6r^2 + 3r - 3$	3	$2r^2 + r - 1$		
$3y^3 + 7y^2 + 2y$				
$8t^2 - 8t - 6$				
$8ac^2 + 14ac + 3a$				

ANSWERS

$3(2r-1)(r+1)$

$y(3y+1)(y+2)$

$2(2t+1)(2t-3)$

$a(2c+3)(4c+1)$

EXPERIENCE III.9.A

NAME ________________________ TIME STARTED TIME FINISHED

A GENERAL RULE IN FACTORING POLYNOMIALS IS TO ALWAYS REMOVE ANY COMMON FACTOR FIRST. IN THIS OBJECTIVE WHAT REMAINS AFTER THE COMMON FACTOR IS REMOVED WILL BE A TRINOMIAL WHICH IS A PERFECT SQUARE. REMEMBER OBJECTIVE III.2. IT WILL HELP YOU IN FACTORING PERFECT SQUARES.

DO YOU KNOW WHAT YEAR THE OLYMPIC GAMES WILL BE IN MONTREAL? TO FIND OUT REMOVE A COMMON FACTOR FROM EACH EXPRESSION AND THEN FACTOR AS A PERFECT SQUARE. YOU SHOULD FIND YOUR ANSWER IN THE RIGHT HAND COLUMN.

_____ $3t^2 + 18t + 27$

_____ $4at^2 - 20at + 25a$

_____ $100at^2 + 80at + 16a$

_____ $9t^3 - 12t^2 + 4t$

1. $3(t+3)^2$
5. $a(10t+16)^2$
6. $t(3t-2)^2$
7. $3(3t-2)^2$
7. $4a(5t+2)^2$
8. $2a(2t-5)^2$
9. $a(2t-5)^2$

ONLY TWO MORE YEARS. WILL YOU BE GOING TO MONTREAL?

EXPERIENCE III.9.B

NAME ______________________________ TIME STARTED TIME FINISHED

CHANCES ARE YOU CAN REMOVE THE COMMON FACTOR CORRECTLY, BUT YOU MUST BE CAREFUL WITH THE TRINOMIAL THAT REMAINS. IS IT REALLY A PERFECT SQUARE? TO FIND OUT YOU CHECK THE FIRST AND LAST TERMS. THEY MUST BE PERFECT SQUARES.

FOR EXAMPLE: $x^2 + 6x + 9$ AND $4m^2 - 20m + 25$ ARE GOOD POSSIBILITIES SINCE x^2, $(+9)$, $4m^2$, AND $(+25)$ ARE ALL PERFECT SQUARES, BUT $x^2 + 6x + 8$ AND $4m^2 - 21m - 25$ ARE NOT BECAUSE NEITHER 8 NOR (-25) ARE PERFECT SQUARES.

THERE IS ONE OTHER THING TO WATCH. EVEN WHEN THE FIRST AND LAST TERMS ARE PERFECT SQUARES, THE MIDDLE TERM MAY CAUSE PROBLEMS.

$x^2 + 6x + 9 = (x+3)^2$ ⟵ A PERFECT SQUARE

$x^2 + 10x + 9 = (x+9)(x+1)$ ⟵ NOT A PERFECT SQUARE

FOR EACH OF THE FOLLOWING REMOVE A COMMON FACTOR AND THEN FACTOR FURTHER ONLY IF IT IS A PERFECT SQUARE. YOU WILL FIND 4 THAT ARE PERFECT SQUARES AND 3 THAT ARE NOT.

G. $am^2 + 2am + a =$

B. $3x^2 - 18x + 9 =$

O. $3z^2 + 30z + 75 =$

A. $p^3 - 3p^2 - 4p =$

O. $9r^3 - 42r^2 + 49r =$

H. $2yr^2 - 10yr + 8y =$

D. $8ax^2 + 40ax + 50a =$

G. $a(m+1)^2$
O. $3(z+5)^2$
O. $r(3r-7)^2$
D. $2a(2x+5)^2$

B.
A. } These are not perfect squares.
H.

EXPERIENCE III.9.C

NAME ______________________________ TIME STARTED TIME FINISHED

REMOVE A COMMON FACTOR AND FACTOR A PERFECT SQUARE.

EXAMPLE: $9ax^2 + 12ax + 4a$

$= a(9x^2 + 12x + 4)$ ← REMOVE A COMMON FACTOR

$= a(3x+2)^2$ ← FACTOR A PERFECT SQUARE

YOU CAN CHECK BY WORKING IN REVERSE.

$a(3x+2)^2$

$= a(3x+2)(3x+2)$

$= a(9x^2 + 6x + 6x + 4)$

$= a(9x^2 + 12x + 4)$

$= 9ax^2 + 12ax + 4a$

NOW TRY THESE THREE:

REMOVE A COMMON FACTOR AND THEN FACTOR THE PERFECT SQUARE.

1. $3x^2 + 30x + 75$
2. $9ay^2 - 24ay + 16a$
3. $8z^3 - 24z^2 + 18z$

DID YOU GET $3(x+5)^2$, $a(3y-4)^2$, AND $2z(2z-3)^2$? IF YOU DID THEN GREAT! IF YOU DID NOT THEN BETTER ASK FOR A BIT OF HELP.

EXPERIENCE III.10.A.

NAME: ____________________ **TIME STARTED:** **TIME FINISHED:**

Do you know how to take out a common factor from a binomial such as $3b^2 - 12$? Can you factor $b^2 - 4$ as the difference of two squares? If you can do both these things you should be able to factor completely $3b^2 - 12$. (Answer $3(b - 2)(b + 2)$.)

Now, find an expression in Column 2 which is equivalent to the binomial in Column 1.

Column 1.	Column 2.
______ 1. $2b^2 - 50$	A. $2(b - 5)(b - 5)$
______ 2. $27 - 3b^2$	B. $2(b - 5)(b + 5)$
______ 3. $9bc^2 - 4b$	O. $b(c - 7)(c + 7)$
______ 4. $18b^2 - 8$	T. $2(3b - 2)(3b + 2)$
______ 5. $bc^2 - 49b$	R. $b(5 - 2c)(5 + 2c)$
______ 6. $4b - 25bc^2$	S. $b(3c - 2)(3c + 2)$
	O. $3(3 - b)(3 + b)$
	N. $b(2 - 5c)(2 + 5c)$
	P. $2(2b - 3)(2b + 3)$

Did your answers give you a mystery word? How did this team get on in the Stanley Cup? Did they beat Chicago ?

EXPERIENCE III 10.B.

NAME: ____________________ TIME STARTED: TIME FINISHED:

Hi there! Welcome back! Still having a tough time? Now, Objective III.10. involves TWO STEPS.

STEP 1: Remove a common factor.

STEP 2: Factor the remaining binomial as the difference of two squares.

Example. Factor $3a^2 - 12$

STEP 1. Remove the common factor 3. So we have,

$$3a^2 - 12 = 3(a^2 - 4)$$

STEP 2. Now we have to factor $a^2 - 4$. So, we have,

$$a^2 - 4 = (a - 2)(a + 2)$$

Therefore $3a^2 - 12 = 3(a - 2)(a + 2)$

How easy can this factoring be? Let's see how you get on with the following:

Factor:

1. $3k^2 - 3$
2. $2t^2 - 50$
3. $rc^2 - 25r$
4. $9bc^2 - 4b$
5. $32t^5 - 8t^3$

Answers:

1. $3(k - 1)(k + 1)$
2. $2(t - 5)(t + 5)$
3. $r(c - 5)(c + 5)$
4. $b(3c - 2)(3c + 2)$
5. $8t^3(2t + 1)(2t - 1)$

EXPERIENCE III. 10.C.

NAME:____________________ TIME STARTED: TIME FINISHED:

John: This factoring bit is a drag.

Don: How come?

John: That crazy Formative Test asked us to factor $12k^2 - 27$ and I blew it.

Don: Well what's the first thing we should look for?

John: A common factor.... which is 3, and if we take it out we get $3(4k^2 - 9)$

Don: Great , mate! What about $4k^2 - 9$?

John: Well, $4k^2$ and 9 are both squares, so $4k^2 - 9$ is the difference of two squares and its factors are $(2k + 3)(2k - 3)$

Don: Right on, John! So what are the factors of $12k^2 - 27$?

John: Would you believe $3(2k + 3)(2k - 3)$?

Don: Good show! Now try to factor these by removing a common factor and then factoring the remaining difference of two squares.

1. $2b^2 - 8$
2. $kb^2 - 9k$
3. $4x^2 - 36$
4. $25bc^2 - 9b$

Answers:

1. $2(b + 2)(b - 2)$
2. $k(b + 3)(b - 3)$
3. $4(x + 3)(x - 3)$
4. $b(5c + 3)(5c - 3)$

How did you get on? If you are having trouble, ASK SOMEONE FOR HELP!

REVIEW SHEET III

THE REVIEW SHEET IS GIVEN FOR TWO REASONS. FIRST, IF YOU MASTERED AN OBJECTIVE EARLY IN THE SUB-UNIT IT MAY HAVE SLIPPED YOUR MIND. SECONDLY, PERHAPS YOU HAD QUITE A BIT OF TROUBLE AND NEED A BIT OF EXTRA PRACTICE. IN EITHER CASE THESE QUESTIONS WILL LET YOU KNOW HOW YOU ARE DOING. GOOD LUCK!

NOTE: THE QUESTIONS ARE NOT IN THE SAME ORDER AS YOUR OBJECTIVES. CAN YOU MATCH THEM UP?

EVALUATE THE FOLLOWING:

1. $(q+5)^2$
2. $(p+7)(p-7)$
3. $(2r-3)^2$
4. $(4x+3)(4x-3)$

FACTOR THE FOLLOWING:

5. $2mn - 2mx + 3yn - 3yx$
6. $ab - ac - db + dc$
7. $4x + 8y + px + 2py$
8. $5a + 10 - ax - 2x$
9. $t^2 - 16t + 64$
10. $m^2 - 16$
11. $16p^2 + 40p + 25$
12. $9q^2 - 4$
13. $3z^2 - 12z + 12$
14. $12a^2 + 8a - 4$
15. $3m^2 - 12$
16. $3x^2 + 21x + 18$
17. $4r^2y + 12ry + 9y$
18. $9w^2t - 25t$
19. $ax^2 - 4ax - 12a$
20. $6mn^2 - 13mn + 6m$

ANSWERS

1. $q^2 + 10q + 25$
2. $p^2 - 49$
3. $4r^2 - 12r + 9$
4. $16x^2 - 9$
5. $(2m+3y)(n-x)$
6. $(a-d)(b-c)$
7. $(x+2y)(4+p)$
8. $(a+2)(5-x)$
9. $(t-8)^2$
10. $(m+4)(m-4)$
11. $(4p+5)^2$
12. $(3q+2)(3q-2)$
13. $3(z-2)^2$
14. $4(3a-1)(a+1)$
15. $3(m+2)(m-2)$
16. $3(x+6)(x+1)$
17. $y(2r+3)^2$
18. $t(3w+5)(3w-5)$
19. $a(x-6)(x+2)$
20. $m(3m-2)(2m-3)$

APPENDIX D

SUMMATIVE AND POSTSUMMATIVE TESTS

SUMMATIVE TEST

THIS TEST IS TO DETERMINE HOW WELL YOU HAVE MASTERED THE MATHEMATICS YOU HAVE STUDIED FOR THE PAST FOUR WEEKS. THERE IS ONE QUESTION ON EACH OF THE 30 OBJECTIVES STUDIED AND YOU ARE TO SUPPLY THE ANSWER ONLY, IN THE SPACE PROVIDED. YOU MAY DO YOUR WORK ON SCRATCH PAPER. TAKE YOUR TIME AND BE CAREFUL. GOOD LUCK!

1. Evaluate $2ab^2 \cdot 3ab^3$
2. Find the missing factor $8m^2t^3 = (2mt)(\underline{\ ?\ })$
3. Find the product $3x^2(y-2x)$
4. Factor by removing a common factor $6rt - 3t$
5. Expand $2a(ac-3ab+2)$
6. Factor by removing a common factor $10y^3 - 8y^2z + 6y$
7. Factor $2m(3q-1) + 3(3q-1)$
8. Expand $(2a+3)(b+2c)$
9. Find the product $(x-2y)(z-x)$
10. Evaluate $(m-t)(p+r)$
11. Expand $(x+3)(x-5)$
12. Evaluate $(2y-3)(3y-4)$

Factor the following trinomial expressions:

13. $t^2 + 6t + 8$
14. $6m^2 + 7m + 2$
15. $r^2 - 7r + 6$
16. $3x^2 - 11x + 6$
17. $a^2 + 3a - 18$
18. $8q^2 + 10q - 3$
19. $c^2 - c - 20$
20. $5m^2 - 7m - 6$

21. Square $(3r+2)$ ________________

22. Factor as a perfect square $z^2 - 12z + 36$ ________________

23. Expand $(t+5)(t-5)$ ________________

24. Factor as the difference of squares $4y^2 - 49$ ________________

Group and then factor the following polynomials:

25. $mt - xt + mr - xr$ ________________

26. $6yz + 4az - 3y - 2a$ ________________

Factor by removing a common factor and then factoring the remaining trinomial:

27. $m^3 - 7m^2 + 10m$ ________________

28. $12y^2 + 21y - 6$ ________________

Factor by removing a common factor and then factoring the remaining trinomial as a perfect square:

29. $9a^2b - 12ab + 4b$ ________________

Factor by removing a common factor and then factoring the remaining binomial as the difference of squares:

30. $3at^2 - 12a$ ________________

POSTSUMMATIVE TEST

THIS TEST IS TO DETERMINE HOW WELL YOU HAVE REMEMBERED THE MATHEMATICS WHICH YOU STUDIED SEVERAL WEEKS AGO. THERE IS ONE QUESTION ON EACH OF THE 30 OBJECTIVES STUDIED AND YOU ARE TO PROVIDE THE ANSWER ONLY, IN THE SPACE PROVIDED. YOU MAY DO YOUR WORK ON SCRATCH PAPER. TAKE YOUR TIME AND BE CAREFUL. GOOD LUCK!

1. Evaluate $3x^2y \cdot 4x^3y$ __________
2. Find the missing factor $6a^3t^2 = (3a^2t)(\underline{\ ?\ })$ __________
3. Find the product of $2m^2(p-3q)$ __________
4. Factor by removing a common factor $8ab - 4b$ __________
5. Expand $3y(yw-2yz+3)$ __________
6. Factor by removing a common factor $8t^3 - 6t^2r + 4t$ __________
7. Factor $3q(2w-1) + 2(2w-1)$ __________
8. Expand $(2m+5)(x+2c)$ __________
9. Find the product $(r-3y)(a-r)$ __________
10. Evaluate $(c-t)(q+z)$ __________
11. Expand $(m+2)(m-7)$ __________
12. Evaluate $(3d-2)(4d-3)$ __________

Factor the following trinomial expressions:

13. $y^2 + 5y + 6$ __________
14. $6n^2 + 11n + 3$ __________
15. $t^2 - 9t + 8$ __________
16. $2w^2 - 7w + 6$ __________
17. $b^2 + 5b - 24$ __________
18. $10r^2 - 13r - 3$ __________
19. $y^2 - y - 12$ __________
20. $3m^2 - 7m - 6$ __________

21. Square $(2x+3)$ ________________

22. Factor as a perfect square $t^2 - 10t + 25$ ________________

23. Expand $(z+4)(z-4)$ ________________

24. Factor as the difference of squares $36x^2 - 49$ ________________

Group and then factor the following polynomials:

25. $ab - cb + ax - cx$ ________________

26. $10rt + 4mt - 5r - 2m$ ________________

Factor by removing a common factor and then factoring the remaining trinomial:

27. $y^3 - 6y^2 + 8y$ ________________

28. $8c^2 + 22c - 6$ ________________

Factor by removing a common factor and then factoring the remaining trinomial as a perfect square:

29. $4m^2t - 12mt + 9t$ ________________

Factor by removing a common factor and then factoring the remaining binomial as the difference of squares:

30. $2xa^2 - 18x$ ________________

ITEM ANALYSIS FOR

ITEM	SUMMATIVE TEST		POSTSUMMATIVE TEST	
	DIFFICULTY	BISERIAL CORR	DIFFICULTY	BISERIAL CORR
1	.82	.71	.89	.62
2	.91	.67	.92	.82
3	.78	.63	.80	.51
4	.69	.75	.76	.85
5	.79	.74	.86	.69
6	.57	.65	.63	.58
7	.65	.70	.77	.88
8	.65	.58	.77	.66
9	.49	.80	.64	.87
10	.57	.81	.62	.77
11	.55	.71	.55	.78
12	.46	.73	.52	.73
13	.83	.92	.84	.77
14	.68	.91	.74	.85
15	.55	.93	.65	.99
16	.53	.93	.56	.95
17	.64	.85	.69	.94
18	.40	.88	.50	.89
19	.43	.90	.56	.92
20	.49	.95	.60	.85
21	.64	.81	.62	.84
22	.68	.75	.68	.84
23	.64	.73	.77	.78
24	.57	.83	.76	.99
25	.77	.82	.84	.84
26	.61	.82	.67	.82
27	.51	.91	.58	.91
28	.36	.90	.46	.82
29	.53	.90	.52	.81
30	.48	.80	.65	.90

N = 152	N = 153
$\bar{X}$ = 18.2	$\bar{X}$ = 20.4
S^2 = 75.9	S^2 = 69.1
K-R 20 = .92	K-R 20 = .92

APPENDIX E

ATTITUDE TOWARD MATHEMATICS TEST

SELF CONCEPT OF ABILITY QUESTIONNAIRE

MASTERY LEARNING OPINIONNAIRE

CLASSROOM OBSERVATION RECORD

ATTITUDE TOWARD MATHEMATICS

Name: ____________________ ____________________ ____________________
(Last Name) (First Name) (Middle Name)

Class: ____________________

At the top of the third page you will notice the statement:

Learning and Doing Mathematics

Below this statement are a series of word pairs. For example:

happy _______, _______, _______, _______, _______sad

You are to react to this statement by placing an "X" in one of the five spaces separating the two paired words. Mark your "X" in the space that best indicates the degree of your feeling toward the statement expressed by the word pairs.

For example:

(a) If your feelings toward Mathematics suggest to you very strongly the idea "happy" then you would mark it as follows:

happy ___X___, _______, _______, _______, _______sad

(b) If your feelings toward Mathematics suggest to you a slightly strong feeling of the idea "happy" then you would mark it as follows:

happy _______, ___X___, _______, _______, _______sad

(c) If your feelings toward Mathematics suggest to you a feeling which is not happy and not sad (neutral) then you would mark it as follows:

happy _______, _______, ___X___, _______, _______sad

(d) If your feelings toward Mathematics suggest to you a slightly strong feeling of the idea "sad" then you would mark it as follows:

happy _______, _______, _______, ___X___, _______sad

(e) If your feelings toward Mathematics suggest to you very strongly the idea "sad" then you would mark it as follows:

happy ______, ______, ______, ______, __X__ sad

If your feelings are neutral about a word pair, or if you feel the word pair is unrelated to your feelings, place the "X" in the middle space.

IMPORTANT: 1. Be sure to mark only one "X" in the spaces for every word pair. DO NOT OMIT any word pairs.

2. Mark down your first feelings when you read the word pairs. We want your true feelings, however do not be careless.

LEARNING AND DOING MATHEMATICS

1.	worthwhile	_____	_____	_____	_____	_____	worthless
2.	not secure	_____	_____	_____	_____	_____	secure
3.	familiar	_____	_____	_____	_____	_____	strange
4.	happy	_____	_____	_____	_____	_____	sad
5.	deadly	_____	_____	_____	_____	_____	lively
6.	hard	_____	_____	_____	_____	_____	easy
7.	failure	_____	_____	_____	_____	_____	successful
8.	student centered	_____	_____	_____	_____	_____	teacher centered
9.	like	_____	_____	_____	_____	_____	dislike
10.	practical	_____	_____	_____	_____	_____	not practical
11.	not creative	_____	_____	_____	_____	_____	creative
12.	useless	_____	_____	_____	_____	_____	valuable
13.	fair	_____	_____	_____	_____	_____	unfair
14.	stupid	_____	_____	_____	_____	_____	smart
15.	fast	_____	_____	_____	_____	_____	slow
16.	unsure	_____	_____	_____	_____	_____	sure
17.	dull	_____	_____	_____	_____	_____	interesting
18.	unimportant	_____	_____	_____	_____	_____	important
19.	tense	_____	_____	_____	_____	_____	relaxed
20.	bad	_____	_____	_____	_____	_____	good
21.	active	_____	_____	_____	_____	_____	inactive
22.	enjoyable	_____	_____	_____	_____	_____	dull
23.	favorite	_____	_____	_____	_____	_____	least favorite
24.	different	_____	_____	_____	_____	_____	usual
25.	boring	_____	_____	_____	_____	_____	interesting

QUESTIONNAIRE.

INTRODUCTION:

We are interested in finding out what young people think about themselves and their school work. you can help us to better understand young people of your age by answering the following questions as honestly as you can.

Please read carefully the directions before you answer. If you have any questions, raise your hand and someone will help you.

The answers you give will be considered as CONFIDENTIAL INFORMATION and will in no way affect your grades. No one will see the answers you give except the research team.

Your help in this study is greatly appreciated.

PLEASE FILL IN THE FOLLOWING INFORMATION

Name: ________________ (Last Name) ________________ (First Name) ________________ (Middle Name)

Class: ________________

Circle the letter in front of the statement which best answers each question.

1. How do you rate yourself in school ability compared with your close friends?
    - a. I am the best
    - b. I am above average
    - c. I am average
    - d. I am below average
    - e. I am the poorest
2. How do you rate yourself in school ability compared with those in your class at school?
    - a. I am among the best
    - b. I am above average
    - c. I am average
    - d. I am below average
    - e. I am among the poorest
3. Where do you think you would rank in your class in high school?
    - a. among the best
    - b. above average
    - c. average
    - d. below average
    - e. among the poorest
4. Do you think you have the ability to complete college?
    - a. yes,definitely
    - b. yes,probably
    - c. not sure either way
    - d. probably not
    - e. no

5. Where do you think you would rank in your class in college?
    - a. among the best
    - b. above average
    - c. average
    - d. below average
    - e. among the poorest

6. In order to become a doctor, lowyer, or university professor, work beyond four years of college is necessary. How likely do you think it is that you could complete such advanced work?
    - a. very likely
    - b. somewhat likely
    - c. not sure either way
    - d. unlikely
    - e. most unlikely

7. Forget for a moment how others grade your work. In your own opinion how good do you think your work is?
    - a. my work is excellent
    - b. my work is good
    - c. my work is average
    - d. my work is below average
    - e. my work is much below average

8. What kind of grades do you think you are capable of getting?
    - a. mostly A's
    - b. mostly B's
    - c. mostly C's
    - d. mostly D's
    - e. mostly F's

9. How important to you are the grades you get in school?

    a. very important

    b. important

    c. not particularly important

    d. grades don't matter to me at all

10. How important is it to you to be high in your class in grades?

    a. very important

    b. important

    c. not particularly important

    d. doesn't matter to me at all

11. How do you feel if you don't do as well in school as you know you can?

    a. feel very badly

    b. feel badly

    c. don't feel particularly badly

    d. doesn't bother me at all

12. How important is it to you to do better than others in school?

    a. very important

    b. important

    c. not particularly important

    d. doesn't matter to me at all

13. Which statement best describes you?

    a. I like to get better grades than everyone else.

    b. I like to get better grades than almost everyone else.

    c. I like to get about the same grades as everyone else.

    d. I don't care about any particular grades.

14. In your schoolwork do you try to do better than others?

    a. all of the time

    b. most of the time

    c. occasionally

    d. never

15. How important to you are good grades compared with other aspects of school?

    a. good grades are the most important thing in school

    b. good grades are among the important things in school

    c. some other things in school are more important than good grades

    d. good grades don't matter to me at all

16. What kind of grades do you try to get in school?

    a. mostly A's

    b. mostly B's

    c. mostly C's

    d. mostly D's

    e. don't try to get any particular grades

Go On To The Next Page

PART TWO

Now, we would like you to answer some of the same questions but this time about MATHEMATICS. CIRCLE THE 'X' UNDER THE HEADING WHICH BEST ANSWERS THE QUESTION.

1. How do you rate your ability in Mathematics compared with your close friends?

I am the poorest	I am below average	I am average	I am above average	I am the best
X	X	X	X	X

2. How do you rate your ability in Mathematics compared with those in your class at school?

I am among the poorest	I am below average	I am average	I am above average	I am among the best
X	X	X	X	X

3. Where do you think you would rank in your high school graduating class in Mathematics?

among the poorest	below average	average	above average	among the best
X	X	X	X	X

4. Do you think you have the ability to do college work in Mathematics?

no	probably not	not sure either way	yes probably	yes definitely
X	X	X	X	X

5. Where do you think you would rank in your college class in Mathematics?

among the poorest	below average	average	above average	among the best
X	X	X	X	X

6. How likely do you think it is that you could complete advanced work beyond college Mathematics?

most unlikely	unlikely	not sure either way	somewhat likely	very likely
X	X	X	X	X

7. Forget for a moment how others grade your work. In your own opinion how good do you think your work is in Mathematics?

my work is much below average	my work is below average	my work is average	my work is good	my work is excellent
X	X	X	X	X

8. What kind of grades do you think you are capable of getting in Mathematics?

mostly F's	mostly D's	mostly C's	mostly B's	mostly A's
X	X	X	X	X

9. How important to you are the grades you get in Mathematics?

grades don't matter to me at all	not particularly important	important	very important
X	X	X	X

10. How important is it to you to be high in your class in Mathematics?

it doesn't matter to me at all	not particularly important	important	very important
X	X	X	X

11. How do you feel if you don't do as well as you know you can in Mathematics?

doesn't bother me at all	don't feel particularly badly	feel badly	feel very badly

___X____________X____________X____________X___

12. How important is it to you to do better than others in Mathematics?

doesn't matter to me at all	not particularly important	important	very important

___X____________X____________X____________X___

13. Which statement best describes your feeling about Mathematics?

I don't care about any particular grade	I like to get about the same grade as everyone else	I like to get better grades than almost everyone else	I like to get better grades than everyone else

_____X_____________X_____________X______________X____

14. Do you try to do better in Mathematics than in other subjects?

never	occasionally	most of the time	all of the time

_X_____________X_____________X_____________X___

15. What grades do you try to get in Mathematics?

don't try to get any particular grades	mostly D's	mostly C's	mostly B's	mostly A's

_____X_________X_______X_______X________X__

MASTERY LEARNING OPINIONNAIRE

Name: ____________________ Class: ____________________

During the past four weeks you have been learning mathematics by Mastery Learning methods.

We would appreciate if you would answer the following questions. If the space provided is not sufficient, please use the back of the page. Your answers will, of course, be CONFIDENTIAL.

Thank you for your cooperation.

Question 1. How is this mathematics different from what you have had before?

Question 2. How do you think we could improve on this type of mathematics?

CLASSROOM OBSERVATION

CLASS ____________________ DATE ______________________

TIME STARTED _____________ TIME FINISHED _____________

TIME	PEER TUTORING	TEACHER -STUDENT	RESEARCHER -STUDENT	OTHER
MIN. 1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

APPENDIX F

DATA GATHERED FOR THE STUDY

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
00121	74	75	77	72	3	086	30	24	29	20	078	21	19	23	18	24	27
00221	35	53	42	46	2	085	25	26	23	21	076	23	24	23	19	11	09
00321	55	41	48	54	3	105	31	28	27	23	117	31	29	28	26	21	29
00421	74	75	77	63	3	072	26	23	21	19	076	24	20	21	15	27	27
00521	21	23	19	15	2	068	16	22	15	18	071	15	20	15	17	02	04
00621	52	91	75	67	5	096	32	25	35	21	099	33	26	32	23	29	27
00721	74	78	79	76	4	091	34	24	31	19	097	34	24	31	20	26	30
00821	48	33	40	28	2	074	21	23	18	21	068	20	25	18	22	06	06
00921	68	81	77	84	5	090	30	28	34	26	075	35	23	35	20	27	27
01021	07	53	20	28	2	096	24	21	24	20	068	22	22	22	19	06	21
01121	52	61	56	31	3	081	25	20	25	17	081	26	20	25	17	15	16
01221	84	72	82	88	3	067	32	22	22	17	059	31	23	20	16	16	22
01321	65	81	75	58	2	072	26	19	26	15	060	28	18	27	16	10	20
01421	61	84	75	28	4	084	27	20	35	23	056	30	20	26	20	15	12
01521	87	99	97	42	5	112	36	32	37	28	080	31	22	33	26	20	25
01621	29	14	19	23	4	107	31	27	32	24	106	33	24	28	23	30	28
01721	23	26	22	28	3	043	21	22	16	18	055	17	24	14	18	07	15
01821	52	91	75	88	4	079	34	25	31	18	060	31	25	32	20	22	24
01921	91	81	90	94	5	103	35	26	34	22	097	34	28	35	22	30	28
02021	68	67	70	46	4	115	31	25	32	22	108	31	26	31	23	21	24
02121	04	41	12	08	2	064	22	22	16	15	066	23	21	30	18	07	14
02221	61	33	48	67	3	094	29	23	23	18	106	29	20	26	17	20	25
02321	84	68	80	84	3	095	27	23	19	17	099	21	24	22	19	27	28

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
02421	84	75	83	67	5	110	40	33	38	29	106	40	33	36	28	21	29
02521	55	33	44	91	3	084	34	22	25	19	079	30	22	26	17	20	24
02661	84	65	79	67	4	096	34	24	32	21	094	34	25	33	22	27	28
02761	87	94	94	76	5	063	33	26	35	21	081	36	24	33	19	30	30
02861	68	94	86	88	4	102	30	25	32	20	098	29	25	30	20	29	27
02961	32	65	46	38	3	083	28	20	25	20	080	27	19	26	18	28	24
03061	77	75	79	80	3	063	27	26	19	19	098	28	26	20	20	30	26
03161	93	94	96	71	4	108	33	26	35	24	105	33	25	32	23	29	29
03261	27	36	25	19	1	096	33	31	27	27	101	33	31	27	26	09	06
03361	79	53	70	21	2	068	28	21	21	16	076	25	21	20	17	03	08
03461	32	86	60	35	3	102	32	30	35	27	094	30	28	31	28	07	10
03561	26	37	29	26	2	085	26	30	25	24	058	24	30	20	25	11	10
03661	87	93	93	97	5	091	36	28	35	25	085	33	29	32	22	28	30
03761	84	99	96	94	5	087	36	19	35	19	082	30	16	28	13	29	30
03861	35	93	68	46	3	072	23	25	24	19	097	27	26	25	24	27	30
03961	23	32	23	15	2	095	28	23	28	19	076	29	23	17	19	03	21
04061	10	26	13	01	2	092	23	23	24	20	058	15	27	16	24	12	17
04161	42	75	58	38	3	099	28	27	31	26	091	24	26	22	21	04	06
04261	48	89	72	58	3	104	33	26	27	19	088	29	22	27	19	27	22
04361	23	96	64	54	3	093	30	31	31	28	109	32	29	27	25	19	17
04461	35	68	50	31	1	086	29	27	29	24	081	29	28	29	23	10	11
04561	42	05	19	46	1	075	23	24	18	22	056	16	28	12	20	00	00
04661	55	49	52	80	4	085	27	26	28	21	090	29	23	29	20	28	28

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
04761	71	30	52	38	3	075	25	24	19	18	071	21	24	17	18	12	20
04861	48	72	60	46	3	089	31	28	26	24	093	32	28	26	23	24	27
04932	38	17	25	31	3	080	29	18	28	17	085	28	21	26	19	20	24
05032	77	17	48	88	3	082	26	27	23	22	070	23	27	23	21	12	10
05132	93	61	83	91	4	098	37	31	33	26	097	35	25	31	23	19	21
05232	38	78	58	50	4	118	36	32	38	28	113	33	28	38	29	28	28
05332	55	84	72	58	4	098	35	31	35	25	096	35	32	34	27	29	28
05432	55	37	46	67	3	089	31	28	24	23	077	31	28	23	21	15	11
05532	77	75	79	46	3	058	24	18	21	15	056	24	22	20	17	11	14
05632	55	57	56	67	4	086	25	24	27	16	043	33	18	22	21	25	27
05732	52	23	36	50	2	091	25	21	22	19	093	27	24	22	20	10	19
05832	61	41	52	63	3	101	32	23	23	16	091	34	27	23	22	15	10
05932	84	96	94	58	3	086	3-	23	29	14	076	37	14	36	16	25	24
06032	93	78	90	94	5	119	36	32	35	28	083	33	18	22	21	24	22
06132	48	61	54	46	4	084	34	23	26	20	073	31	26	24	22	15	14
06232	18	32	24	06	3	092	26	26	25	22	105	26	26	28	20	09	19
06332	52	91	75	19	4	086	36	22	36	24	118	39	25	39	22	22	26
06432	16	57	30	02	2	044	15	22	13	19	061	20	20	17	18	05	18
06532	32	33	30	13	3	097	30	27	29	21	088	26	25	25	21	10	10
06632	35	49	40	42	4	089	29	24	28	21	089	29	27	27	22	20	21
06732	68	49	60	54	3	059	19	16	15	14	086	22	18	25	17	22	25
06832	82	75	82	94	5	107	40	30	38	26	104	39	30	36	26	27	26
06932	18	17	15	19	3	081	19	27	19	21	090	19	26	19	21	07	07

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
07032	87	68	82	76	2	072	30	27	20	21	086	29	26	25	21	08	06
07132	23	57	36	17	3	092	26	29	29	24	104	24	31	27	24	22	30
07252	65	81	75	67	4	099	31	26	30	23	107	30	25	29	22	23	29
07352	16	41	24	01	3	089	23	18	26	16	056	19	15	23	11	08	20
07452	35	86	62	21	3	097	27	25	28	24	090	28	24	28	18	23	29
07552	42	49	44	21	4	103	27	26	31	23	111	31	24	32	24	23	24
07652	35	49	46	50	1	055	25	21	18	19	054	23	21	19	19	10	13
07752	65	20	42	26	3	084	29	25	27	20	068	35	23	26	20	14	19
07852	42	72	56	54	3	092	30	25	30	22	097	32	24	32	20	28	24
07952	18	03	08	42	1	062	23	23	12	19	078	21	23	14	18	05	10
08052	94	91	95	97	5	109	32	32	32	26	113	27	32	27	25	28	30
08152	65	68	68	84	3	101	30	26	28	23	118	29	25	27	24	29	29
08252	48	20	32	67	2	103	28	22	28	19	070	25	21	23	16	09	08
08352	35	68	50	28	2	101	29	24	26	18	081	28	21	25	19	20	14
08452	23	45	30	28	3	083	24	26	25	22	071	26	24	26	20	13	12
08552	55	53	54	94	3	972	23	25	20	19	073	25	24	24	18	10	26
08652	20	37	29	54	3	085	24	21	24	18	083	26	20	24	17	17	20
08752	97	98	99	84	5	109	35	26	33	23	106	32	25	32	21	22	26
08852	29	72	48	63	4	097	31	26	31	23	099	31	24	31	22	30	30
08952	91	78	88	97	3	049	36	16	24	14	062	35	19	19	18	05	12
09052	38	49	42	58	4	092	27	24	26	20	086	28	23	27	22	23	26
09152	55	61	58	23	3	097	28	21	27	20	098	24	23	20	20	18	20
09252	74	45	62	63	3	080	30	26	25	17	081	33	27	25	18	19	23

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
09352	55	81	60	84	3	098	29	25	32	24	102	30	25	30	22	21	20
09413	18	23	17	28	3	054	22	20	19	17	073	21	21	22	15	23	28
09513	91	98	97	76	4	101	32	31	36	29	100	33	28	36	25	28	28
09613	29	20	22	21	1	055	16	22	14	19	063	17	25	15	21	08	02
09713	84	97	95	46	5	111	32	31	36	28	113	36	29	36	26	30	30
09813	32	68	48	63	4	091	28	29	27	25	096	32	28	30	23	21	25
09913	45	17	29	38	1	074	26	20	22	17	048	20	18	22	14	15	21
10013	68	81	77	72	3	091	34	27	33	24	082	33	26	33	23	15	19
10113	38	14	24	26	3	102	29	21	27	20	086	29	24	31	19	12	11
10213	52	26	38	28	1	056	23	21	12	15	044	20	19	09	15	00	01
10313	45	30	36	58	4	081	26	23	23	20	057	22	24	21	22	05	16
10413	71	72	73	58	2	088	23	20	23	18	085	26	20	22	16	07	09
10513	48	91	73	50	3	092	26	24	29	20	071	26	19	26	15	13	19
10613	16	20	15	13	2	068	19	23	18	17	059	12	23	15	17	06	04
10713	91	78	88	84	4	100	30	23	25	19	097	29	23	26	21	28	29
10813	16	53	29	46	3	083	27	27	26	21	089	28	28	26	23	18	09
10913	48	63	56	42	4	099	29	26	27	20	081	31	26	28	20	13	27
11013	48	33	40	50	2	073	26	26	19	19	033	21	23	20	19	01	03
11113	82	96	93	72	3	066	25	18	27	17	063	20	20	20	17	27	28
11213	74	81	80	58	4	112	36	30	33	27	058	39	30	31	24	21	17
11313	26	33	27	15	3	088	27	25	22	22	073	22	26	22	22	11	15
11413	61	57	60	50	3	092	35	28	26	19	067	35	28	25	19	22	25
11513	02	05	01	03	2	079	25	22	24	20	056	18	20	18	17	00	04

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
11613	18	09	11	19	2	097	27	17	25	22	085	29	21	29	20	23	22
11713	99	81	95	96	3	065	37	28	24	17	059	37	27	19	15	04	09
11813	52	89	73	13	5	119	38	33	40	29	121	38	33	40	29	29	30
11913	55	49	52	54	4	099	31	23	31	24	094	33	22	33	25	28	26
12044	61	75	70	54	2	096	28	21	25	19	103	30	27	27	22	15	26
12144	65	89	80	76	5	119	38	32	38	29	111	35	31	35	27	27	29
12244	99	84	96	88	5	107	37	32	36	27	100	32	32	30	28	26	29
12344	45	49	46	58	3	114	29	22	28	16	112	29	22	23	18	26	28
12444	26	23	22	23	2	080	27	25	18	20	079	25	26	23	21	17	20
12544	94	97	98	84	5	106	35	24	33	22	098	35	23	35	23	30	30
12644	58	72	66	38	4	107	28	22	31	22	105	27	23	27	24	23	29
12744	61	20	40	67	3	082	21	25	19	17	059	25	24	18	17	21	23
12844	55	81	70	31	4	085	27	19	31	18	089	26	20	30	19	29	24
12944	38	30	32	54	3	050	28	20	17	16	046	29	20	20	16	28	29
13044	58	53	56	63	3	100	26	27	21	23	064	27	25	18	19	12	11
13144	09	37	16	10	2	102	25	21	25	19	107	26	21	27	18	24	26
13244	32	33	30	50	2	061	28	23	16	18	055	27	23	16	17	18	19
13344	09	30	13	17	3	073	22	25	22	21	070	24	23	24	18	10	16
13444	38	84	62	54	4	100	30	29	26	23	102	30	29	33	26	23	29
13544	26	37	29	35	2	083	27	24	26	22	074	25	22	24	19	13	14
13644	18	41	25	31	3	076	28	22	24	19	085	26	18	26	16	25	29
13744	82	78	83	88	4	092	29	30	24	27	084	28	30	30	25	26	25
13844	16	89	50	54	4	095	33	28	32	23	100	31	27	33	24	30	30

X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
13944	42	33	36	38	5	113	33	30	32	27	117	31	27	34	27	30	30
14044	84	45	70	63	4	109	31	23	31	24	112	35	15	34	12	28	27
14144	68	53	62	50	3	090	27	21	27	18	090	31	23	30	20	25	25